Building Our Own Expertise - Japanese Lesson Study in Adult Basic Education

A Case Study

by

Eric Appleton

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First reader: Dianne Ramdeholl

Second reader: Betty Hurley

Abstract

New challenges for mathematics instruction in adult basic education require professional development to help educators teach more difficult content. Japanese Lesson Study has been an influential method of supporting improvement of mathematics teaching worldwide. By adapting a successful model, this case study of a lesson study cycle in adult basic education attempts to capture the experience of using practitioner research to improve the teaching of mathematics to adults as preparation for high school equivalency exams. In order to change teaching culture, teachers should be supported in using their classrooms as the source of knowledge for improvement.

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Acknowledgments

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Introduction

> "Research lessons help you see your teaching from various points of view… A lesson is like a swiftly flowing river; when you're teaching you must make judgments instantly. When you do a research lesson, your colleagues write down your words and the students' words. Your real profile as a teacher is revealed to you for the first time."
>
> (Japanese teacher quoted by Catherine Lewis and Ineko Tsuchida in "A Lesson is Like a Swiftly Flowing River," 1998)

This research project took place in an adult learning center that is part of the City University of New York (CUNY) Adult Literacy/HSE/ESOL Program, which provides a range of basic skills classes to adults without a high school degree, from adult literacy (or beginning reading) classes to HSE classes (for students who are preparing to take a high school equivalency exam). In New York City, adult learning centers are run by different entities, including CUNY, the New York City Department of Education, public library systems, and community-based organizations (Stoneman-Bell & Yankwitt, 2017). Using city, state and federal funding for adult basic education, these adult learning centers provide free classes in Adult Basic Education/High School Equivalency (ABE/HSE) and English for Speakers of Other Languages (ESOL).

ABE/HSE classes are generally broken into the broad subjects of reading/writing and math at different levels of instruction starting from about a 6th grade reading level as determined by the Test of Adult Basic Education (Mandated Tests, 2017). ABE/HSE teachers are often expected to teach a range of subjects, including reading, writing, social studies, science and math. The majority of students who attend ABE/HSE classes in New York City have the goal of passing the Test of Adult Secondary Completion™ (TASC) high school equivalency exam in order to receive a high school diploma issued by New York State (What is the HSE/TASC™ Test?, 2017).

Teaching in Adult Basic Education

My experience as a teacher, program director and staff developer is relevant to this study. For the past 18 years I have worked in a variety of roles in adult basic education. From 1999 until 2013 I taught at the Fortune Society, a social service agency that supports people coming home from prison. After a few years of working as a computer technician, I started volunteering

there in a beginning reading and writing (literacy) class, and eventually was hired to teach computer classes. After teaching computer classes for a few years, I became the teacher/coordinator for ABE/HSE classes and later was promoted to be director of the adult basic education program. Throughout my tenure at the Fortune Society, in addition to other responsibilities, I continued teaching different classes, including reading/writing, mathematics, ESOL and technology at all skill levels. Since 2014 I have worked as a math and science professional development coordinator for the CUNY Adult Literacy/HSE Program. In my current work as a staff developer, I research, write, and share curriculum, run workshops and generally support teachers and administrators.

One of my goals while completing the Masters of Arts in Adult Learning (MAAL) at Empire State has been to overlap my academic study as much as possible with staff development, as a way of enriching my work with a theoretical basis and connecting academic study to practice. My final project in lesson study is an example of how my study in MAAL allowed me to do long-term planning and reflection on my role as a supporter of adult education.

When I first started volunteering as a reading instructor in adult literacy, I remember being overwhelmed thinking about the knowledge I would need to do my job well. People like my first supervisor and mentor, Rachel Herzing, talked with me and passed me books like *The Pedagogy of the Oppressed* (Freire, 1996) and *Other People's Children* (Delpit, 1995) to help me understand the social and educational context of adult literacy. What pedagogy would I use? What measure of phonics and whole language? What political understanding should I have of the adult classroom? What was my role as a young white man with students of color, many of whom

were older than me and had very different life experiences? How would I manage a classroom of mixed ages? I was excited, but terrified.

In hindsight, I realize that Rachel and others helped me join a community of practice (Lave, 1991) with other professionals. I started as an assistant in a beginning literacy class helping Marylyn Gore, a long-time literacy volunteer. I learned from watching Marylyn work with the class, helping in small ways until I eventually took over the class when she was unable to come. My introduction to teaching was similar to an apprenticeship. As Lave describes (1991), I started in a secondary role ("legitimate peripheral participation") and slowly took on the central tasks of teaching.

After being hired as a full-time teacher at the Fortune Society, I was lucky to be a part of a community of talented, dedicated teachers who were supportive, but I didn't always feel that I could be successful in the classroom. I was expected to teach math, for one thing. I had been a mediocre math student and hadn't looked at a textbook since taking a basic liberal arts college requirement I barely passed. I realized later that this wasn't an unusual background for math teachers in adult basic education (Ginsburg, 2011). Many of us come from other careers. We were computer technicians, taxi drivers (Garvey, Gordon, Kleinbard, & Wasserman, 2013) or worked in the post office. We may have been drawn to this work because of our belief in social justice, but now we have to teach a range of content that we may not know well. As teachers in adult basic education classrooms, we are generalists, teaching history, literature, writing, science and math. In order to be successful, our classrooms have to be a place where we can continue to learn and develop the skills needed to teach all this material.

At the Fortune Society in the 2000s, we had as many as eight full-time teachers in our adult basic education program, a situation I later learned was highly unusual, since most adult education teachers work part-time. Working with experienced teachers provided opportunities for talking about teaching formally and informally outside of class. In 2000, John Gordon, formerly from the Open Book (Ramdeholl, 2011), replaced Rachel as the education director at the Fortune Society. Along with a wealth of experience in adult literacy and organizing, John brought a belief in the workplace as a place for teachers to learn and improve their practice. John became a mentor to all of us. Under his leadership, we organized meetings as a place to learn from each other. Once a month, we spent two hours working to deepen our understanding of teaching and learning. (At the time, we were allowed to count these program meetings towards mandatory professional development hours required by the New York State Education Department. Recent rule changes make it more difficult to count self-organized professional development.) In preparation for teaching practice meetings, we read articles, looked at student writing and talked about teaching approaches. For example, we read the book *Reading For Understanding* (Schoenbach, Greenleaf, Cziko, & Hurwitz, 1999), about an apprenticeship approach to reading instruction, which influenced our literacy instruction for years. It was a warm and intellectually stimulating environment and, as a new teacher in adult education, this support was essential.

Over the years, our teacher practice meetings came and went. Sometimes, the workload would be too much. We postponed meetings because of urgent program business. Eventually, we tried other ways of bringing teachers together. We created teacher groups that would meet every other week to share lesson plans and curriculum ideas. They would report out to the larger group

once a month. There were also teaching conversations that happened between teachers and supervisors in individual meetings, but we struggled to make these meetings happen regularly and not be consumed by reporting, program concerns, or classroom management issues.

One of the regrets I have about my work at the Fortune Society has to do with classroom observation and teacher development. We tried establishing a practice of classroom observation and feedback. As a lead teacher in our youth classes, I videotaped my classes and participated in workshops with other teachers to talk about the instruction. One other teacher volunteered for a feedback session after she videotaped her class. However, we weren't able to continue the practice with other teachers. Part of the difficulty probably had to do with a power imbalance, since as a director, I was also responsible for evaluating teachers' performances. In hindsight, I realize that it wasn't realistic to expect teachers to volunteer to be observed in that situation. We didn't have a culture of observation, co-teaching or of opening our classrooms to other teachers. We also lacked a process to help teachers feel comfortable inviting other teachers into their classroom.

Our most successful professional development was grounded in student thinking (or our experience thinking as students). For example, we sometimes read student writing as a group in order to create norms for a writing rubric or learn how to give written feedback to students. I felt that these concrete materials from the classroom gave us the best opportunity for meaningful conversations about teaching decisions. Participating in reading circles and doing math together also reminded us of the experience of being learners.

I was often frustrated when our conversations about teaching remained abstract and didn't necessarily improve our teaching, but I didn't know how to change what we were doing. We talked about concepts such as student-centered classrooms, constructivism or education for social change, but I felt that we didn't get close enough to the particular details of how these ideas are put into practice in the classroom. I felt increasingly concerned that we were not improving our teaching skills or knowledge of student learning. In looking back, I think our biggest problem was that we didn't have a clearly defined model of teacher development. In hindsight, I believe we would have been more successful if we had used a structured approach that could have helped us focus on the concrete experiences of teaching and learning in our classrooms. I wish I had had a better understanding of lesson study at the time, because I believe it holds the promise of connecting reflection to student-centered classrooms, while respecting teachers' knowledge and choice.

If we want to change teaching, teachers need opportunities to reflect on specific choices at the level of instruction. This is naturally going to be a conversation about the design of activities, about planning for instruction, about how we facilitate discussion and respond to things that happen in the moment. Conversation at the level of philosophy of teaching is useful in setting the frame, but it doesn't necessarily help teachers make their classrooms more successful. Teaching is a skill, not just an orientation. It would be absurd to think that a carpenter could improve her craft by discussing carpentry generally with other carpenters involved in different kinds of projects (cabinets, furniture, houses). To improve skill at a craft requires a process

where professionals plan a project, implement the project, look carefully at what worked and what didn't, consult with other professionals about concrete details and try again.

Of course, orientation is important. I believe we should work towards equity in our classrooms. I believe in teaching for liberation (Freire, 1996). I believe that students should have the opportunity to reflect on and make sense of the content of instruction. I also know that effective teaching doesn't magically happen because we have the correct orientation. We still have to understand the subject we teach. In fact, we have to understand it more deeply than if we were just going to lecture or deliver the content through a version of the banking method (Freire, 1996). We have to also understand how students will learn the material and then construct activities that will invite them to participate in a constructive, engaging way (Ball, et al, 2005), along with a plan for responses in the moment (how will I respond to this question or that confusion?) and in follow-up (how can we return to a student's question or misunderstanding to push the whole class to a new place?).

When I started working as a professional developer with adult basic education teachers around the city, I realized that I had been extremely fortunate to have full-time employment as a teacher along with the mentorship of other teachers. Most of the teachers I work with in my current role are part-time, paid by the hour and work without benefits. It's an extremely difficult way to make a living. The work is challenging, the pay is abysmal, and the resources are minimal. To make it worse, part-time teachers generally do not have the informal supports I had as a beginning teacher.

Teaching can be complicated, intellectually stimulating work, if we have the tools we need and some chance of success with our students. Without support from other teachers, it can be exhausting and demoralizing. We create lesson plans that have been written a hundred times. We make mistakes again and again trying to blindly figure this thing out. Desperate for information, we attend workshops on various topics that may or may not connect to our classroom and don't necessarily add up to an approach that can help us prepare for our students' reactions to what we bring. And we try to keep our spirits up when our plan for the day flops.

Teachers do best with the mentorship of more experienced peers with whom to share resources and talk about teaching. Teachers who stay in adult basic education do it because they love the students (Redmond, 2014) and are committed to being effective in their teaching, since they know their students have often made great sacrifices in order to come back to school.

The Common Core Comes to Adult Basic Education

Previous to 2014, the General Educational Development® (GED) exam was used throughout the United States to determine the basis for high school equivalency diplomas issued by the states. In 2014, the replacement of the previous version of the GED exam with a new, more expensive test prompted New York State to switch to the new TASC exam (Hilliard, 2017). Other states decided whether to use one of three tests, the new GED, the TASC or the Hi-SET or a combination of the three (Move Over G.E.D, 2013). All of these new exams are based on the College and Career Readiness Standards (CCRS), which were themselves drawn from the Common Core Curriculum Standards (CCSS) (Pimentel, 2013). The CCRS and the CCSS include similar content standards, key shifts in teaching practice and standards for

mathematical practice that are intended to be woven through instruction (2013). The publishers of these exams argued that the tests would be more rigorous and would narrow the gap between high school equivalency demonstration and college preparation, such that HSE test passers would be more college and career-ready that they had been in the past.

These new tests have made a hard job harder, but teachers in adult basic education have not been universally opposed to shifts in practice and the use of new standards (Redmond, 2014). For example, consider the first sentence describing a CCRS instructional shift towards focus: "Generally speaking, instructors need both to narrow significantly and to deepen the manner in which they teach mathematics, instead of racing to cover topics" (Pimentel, 2013, p. 44). This vision of instruction is not inconsistent with a history of teaching in adult basic education that focuses on depth and coherence, rather than breadth (Garvey, et al, 2013). The Standards for Mathematical Practice (2013), including "Make sense of problems and persevere in solving them," are also consistent with established teaching practice in adult basic education. Our goal is not simply to help students pass the HSE exam, but also to become excited about learning, become confident in their ability to learn, and plan for transitions to college and career. All of these goals require teaching for understanding.

However, the implementation of the CCRS has been problematic, to say the least (Redmond, 2014; Hilliard, 2017). Like previous test-based accountability models, it has left less room for teachers to explore "approaches that engage students as active participants in their own learning" (Garvey, et al, 2013). There is a fear that, antithetical to the stated aims of the standards, the emphasis on fact-based understanding of content will pressure teachers to move towards test preparation (Redmond, 2014), rather than education based on investigation and

in-depth understanding of subject matter. The content on the math section of TASC Readiness Tests (TASC™ Readiness Assessment 5 Test Booklet, 2016) shows that the test is much more difficult than the old GED, requiring broad and specific knowledge of subject matter, as well formal definition-based understandings of mathematics. The TASC emphasizes different skills and knowledge than what was required on the previous version of the GED exam. For example, more than half of the mathematics section of the TASC exam assesses knowledge of functions and algebra. Minimal knowledge of algebra and functions was required to pass in the previous version of the GED.

We know from anecdotal information and quantitative data that the math section of the exam is holding down pass rates and preventing our students from moving on (Hilliard, 2017). Math teachers report that their students walk out of the exams when they see the math test, thinking that there is no way they will pass it (Redmond, 2015). As a result of the change to a new exam, fewer students have taken the TASC and even fewer students have passed. According to Hilliard (2017), 46% fewer people took the TASC in 2015 than took the GED in 2010, possibly because they have been scared off by its difficulty. In 2010, about 47,000 people took the GED in New York State. In 2015, only about 25,000 took the TASC test. In 2010, the New York State overall pass rate was 60%. In 2015, it had dropped to 53%. Few people are taking the HSE exam and even fewer than previously are passing. When students fail the exam these days, it is usually because of math. There was a 15% drop in the pass rate on the math section of the TASC between 2010 and 2015 (Hilliard, 2017).

In response to the challenges posed by the new HSE exam and with the support of the New York State Education Department and the New York State Department of Labor, the

CUNY Adult Literacy/HSE/ESL program published the CUNY HSE Curriculum Framework (Brandt, Leece, Trushkowsky, & Appleton, 2015) covering social studies, science and math, and made it freely available to teachers through downloadable pdf format. The math section of the CUNY HSE Curriculum Framework ("the CUNY Framework") primarily covers functions and algebra and works as a structure that could be adopted more broadly for adult education math classes. This lesson study research project used the CUNY Framework as the organizing curriculum around the content of the functions and algebra.

Increasing the Possibility of Success

In my early career, because of lack of knowledge and experience and because I was rushed for time, I often taught lessons that were poorly planned and executed. Of course, I also had the experience of teaching successful lessons, with evidence that students were engaged and learned what I hoped they would learn. I want that experience for teachers with whom I currently work. I want to help increase the chances that more teachers in adult education have positive experiences and, most importantly, that our students are in classrooms with teachers who are knowledgeable, prepared and skillful. The experience of success creates hope for teachers and students. We need more hope. Without the possibility that we can get better at this, the work is just too difficult.

Teaching mathematics for high school equivalency is complicated work, made more difficult by the introduction of Common Core standards into the HSE exams. Adult students join our classes with a variety of previous experiences in mathematics, some positive, often negative. Students are in class for a relatively short period of time, compared with the number of hours of instruction in high school. Many of our students weren't successful previously and have put a lot

of pressure on themselves to succeed this time. Our teachers feel the pressure as well (Redmond, 2014). In order to be successful teaching math to adults, they need substantial mathematical content knowledge (Ball, Hill, & Bass, 2005; Harel, 2008), knowledge of effective instructional strategies, and appropriate and engaging resources for teaching adults, not to mention a large measure of empathy and love for their students, and the ability to keep teaching in a field that generally does not pay a living wage.

As Lampert (2001) explains, "Doing mathematics requires high-quality tasks within a coherent curriculum, the mathematical, and pedagogical knowledge to implement these tasks skillfully, and the disposition and skill to elicit, analyze, and respond to student thinking, among many other things" (cited in Takahashi, Lewis, & Perry, 2013). Properly responding to the additional challenge of HSE exams based on the Common Core requires professional development processes that respect teachers, build from their knowledge and focus on what works with adult students. Clearly, lesson study as a practice for instructional improvement can't address all of the challenges above, but it has been shown to help develop content knowledge, refine instructional strategies, and honor classroom teachers for their work (Lewis & Tsuchida, 1998). Its efficacy is yet to be shown in adult education. This project explores lesson study's usefulness as a support for math teachers in adult basic education.

Research Questions

The case study addresses the following questions:

- How does lesson study support adult education teachers' content and pedagogical knowledge?

- How can lesson study help adult education teachers feel better able to help students prepare for new HSE exams?
- How does lesson study affect adult education teachers' beliefs about students' abilities?
- How does lesson study affect adult education teachers' conception of themselves as professionals?

Literature Review

Professional Development in Adult Basic Education

New math teachers in adult education don't usually come "with the skills and knowledge needed to design engaging, effective instruction" (Ginsburg, 2011). Very few teachers, even those who participate in math teachers' circles, have a background in mathematics or math education (Appleton, Farina, Holzer, Kotelawala, & Trushkowsky, 2017). Most adult education teachers relearn middle and high school math while learning how to teach it. In order to improve mathematics instruction in adult basic education, teachers need sustained opportunities to learn mathematics and practice instructional approaches. Adult basic education teachers recognize that professional development, instructional materials and opportunities to experiment are required to respond to challenges created by new content standards. As a teacher interviewed by Redmond (2014) explained, "We also need time to try out new curriculum… learn the material ourselves, create curriculum that uses a different model than any we've taught before, try it out, and make it effective for test prep all immediately."

For programs receiving state funding, the New York State Education Department requires 14 hours of professional development training yearly. Teachers in New York City generally

complete these professional development hours through attending workshops offered through the New York City Regional Adult Education Network (NYC Regional Adult Education Network, 2017) or the professional development office at the CUNY Adult Literacy/HSE/ESL Program. A recent sampling of the NYC RAEN's web site shows a range of workshops from introduction to math and science on the CUNY Framework, training to proctor language exams, and strategies for the low-level social studies classroom (NYC Regional Adult Education Network, 2017). The NYSED/CUNY Teacher Leader Project is an example of ongoing professional development for teachers (NYSED Teacher Leaders, 2017), though this opportunity is available to a small fraction of the math teachers in adult education across New York State. The New York City Math Exchange Group (Brover, Deagan and Farina, 2000) and the New York City Community of Adult Math Instructors (Appleton, et al, 2017) are two examples of self-organized, peer-led professional development in mathematics. The workshops given through these math circles have provided learning experiences in mathematics to hundreds of math teachers, but are not recognized as professional development hours for the purposes of fulfilling state mandates. There is clearly a need for sustained professional development that provides structure for planning and teaching new math content (Hilliard, 2017). One-off workshops don't provide continuity needed in development of content knowledge and teaching strategies. Furthermore, changing teaching culture in adult basic education will require something different than individual workshops (Hiebert & Stigler, 2004).

Teaching Through Problem-Solving in Japan

> *"No matter how kindly, clearly, patiently, or slowly teachers explain, they cannot make students understand. Understanding takes place in the students' minds as they connect new information with previously developed ideas, and teaching*

> *through problem solving is a powerful way to promote this kind of thinking… understanding occurs as a byproduct of solving problems and reflecting on the thinking that went into those problem solutions."* - Diana Lambdin (2003)

During the TIMSS Video Study (1997), Stigler and Hiebert were surprised to find that Japanese teachers present problems to students without first explaining how to solve them. In other words, students solve problems not to apply what they have already been taught but to learn new mathematics (Burghes & Robinson, 2010; Takahashi, 2008; Takahashi, Lewis & Perry, 2013). In Japan, mathematics lessons are often structured in order to pose questions to students who must use prior knowledge to solve problems and complete tasks, as opposed to the tendency for teachers in American schools to use their teaching time to explain steps and procedures (Kotelawala, 2012). American teachers almost always demonstrate procedures before asking students to practice them (Stigler & Hiebert, 1997; Stigler & Hiebert, 1999).

A problem-solving approach to teaching mathematics in Japan traces its roots to the textbook, *Teaching Elementary School Mathematics through Problem Posing*, written by Jingo Shimizu in 1924 (Isoda, 2010). Japanese teachers have been influenced more recently by the National Council of Teachers of Mathematics (NCTM) 1986 standards (Fernandez & Yoshida, 2004; Romberg, 1993). They have then used lesson study as a way to make problem-solving "the focus of school mathematics" (McDougal & Takahashi, 2014).

In a Japanese problem-solving math classroom, teachers have four main roles:

- *hatsumon* - asking key questions at certain points of the lesson
- *kikan-shido* - walking around to students' desks, monitoring their problem-solving process and strategies

- *neriage* - "polishing" and ordering student solutions for comparison and contrast in whole-class group discussion
- *matome* - final summary to connect mathematical ideas among the various solutions (Lewis, 2011; Shimizu, 1999; Takahashi, 2006).

As part of a problem-solving approach using open-ended mathematical problems, the practice of *neriage* uses whole-class discussion to develop new mathematical ideas from comparing student strategies presented to the class (Inoue, 2011). The effectiveness of this method is based on seeing connections between different solution strategies and building consensus about which tools work best for different situations (Fernandez & Yoshida 2004). Teaching through problem solving "requires a confluence of supports, including: well-designed problems; knowledge about the mathematics of the problem and about likely student thinking; and coordinated development of instructional strategies to support presentation, discussion, and revision of student ideas" (Takahashi, Lewis, & Perry, 2013).

It may be surprising to see that the CCRS Mathematical Practices fit well with a Japanese view of teaching through problem-solving, but "Make sense of problems and persevere in solving them" is the first practice in the CCRS (Pimentel, 2013). The expectation is that students will learn through problem-solving, not just practice skills with problems after learning. Japanese teachers expect struggle and look for ways to highlight mistakes as an instructional strategy. They believe that students learn best by attempting to solve problems themselves even if that involves making mistakes (Stigler & Hiebert, 1999). This is summed up by the quote, "mistakes can be treasures" (Perry & Lewis, 2009).

What is Lesson Study?

First used in the 1880s, *Jugyō kenkyū* (lesson study) has been used broadly in Japan since the 1980s (Isoda, 2010; Lewis, 2000) and in K-12 schools in the United States more recently (Takahashi & Yoshida, 2004). Lesson study is a process of instructional improvement (Fernandez & Yoshida, 2004; Perry & Lewis, 2009; Stigler & Hiebert, 1999) through practitioner research in which teachers get together to plan, observe and discuss lessons (Lewis, 2002a; Takahashi & Yoshida, 2004). The process increases teacher knowledge with the goal of improving learning in their classrooms. Small groups of teachers meet to plan "research lessons," collect data through class observations and debrief with observers and the presenting teacher (Fernandez & Yoshida, 2004). As compared to traditional research, the lesson plan can be considered analogous to a research proposal, while observing the lesson and debriefing can be compared to data gathering and analysis of the data, respectively (Fujii, 2014). As Murata (2011) writes,

> Lesson study places teachers at the center of the professional activity with their interests and a desire to better understand student learning based on their own teaching experiences. The idea is simple: teachers organically come together with a shared question regarding their students' learning, plan a lesson to make student learning visible, and examine and discuss what they observe. Through multiple iterations of the process, teachers have many opportunities to discuss student learning and how their teaching affects it.

Used for more than a century (Isoda, 2010), lesson study is the major form of professional development in Japan (Lewis, 2002a; Murata & Takahashi, 2002) and has been used

all over the world since the early 2000's (C. Lewis, personal communication, May 2, 2017). Lesson study is the primary means for teachers to learn how to teach new subjects in which they don't have training, along with ways to implement curricular reform (Lewis, 2002a). Lesson study conferences in Japan involve large research lessons and discussions afterwards (Stigler & Hiebert, 1999). A survey of 2500 Japanese schools revealed that almost every school uses lesson study for professional development (Chichibu & Kihara, 2013).

In Japan, lesson study is generally part of a school-based training initiative called *konaikenshu* (in-school training). As part of *konaikenshu*, the faculty chooses a broad-based goal for students in their school, based on the school's mission statement. For example, one school's goal included fostering a group of students that listens, talks, understands, and helps each other (Fernandez & Yoshida, 2004). Another school's goal was for their students to "think autonomously, invent, and learn from each other" (2004). Multiple lesson study groups in a school typically work on the same research theme. Lesson study groups work to achieve the *konaikenshu* goals through specific lessons in the classroom, so the planning of research lessons takes into account social and developmental goals as well as academic ones. The practice of reconnecting to the school's mission statement can keep these ideas alive. As an American teacher said, "Lesson study gives guts to a mission statement, makes it real and brings it to life" (Lewis, Perry, & Hurd. 2004).

A typical lesson study process can be broken down into the following steps (Fernandez & Yoshida, 2004; Lewis, 2002a; Perry & Lewis, 2009; Takahashi & Yoshida, 2004), with 20 or

more hours of planning meetings, observation and debriefing for an entire lesson study cycle (Burghes & Robinson, 2010):

1. A small group of three to six teachers plans a research lesson through goal-setting, development of activities and anticipation of student responses.
2. One teacher teaches the planned research lesson while other members of the group observe and collect data on student thinking, learning, engagement, etc.
3. The group meets to reflect on what was observed during the research lesson.
4. The group revises the research lesson.
5. A different teacher from the group teaches the revised version of the research lesson in his/her classroom while other group members observe.
6. The group reflects on the observation of the revised research lesson and the lesson study process. (Revision and reteaching of the lesson is more common in the United States than it is in Japan. In Japan, teachers will typically discuss what the next lesson should be (C. Lewis, personal communication, May 2, 2017).)

Because the teachers in the lesson study group have planned the lesson together, the focus of observation is on the lesson they planned, rather than being on the teacher. The goal of observation is collection of evidence of student thinking and a greater understanding of the learning process. This is very different from how observation generally occurs in American classrooms, in which teachers are observed by administrators as part of formal evaluation. A final research lesson observation and debriefing may be attended by teachers and administrators

from throughout the school, as well as educators from other schools, though the focus still remains on the students and the lesson, not the teacher who led the class.

When a lesson study group meets for the first time, they consider their research theme. As Lewis writes, "lesson study is *not just about a single lesson*, but about the teaching of an entire unit and subject area, and indeed, about student development more broadly" (italics in original, 2002a). A common set of questions for this discussion follows: "Ideally, what qualities do we hope our students will have when they graduate from our school...? What are the actual qualities of our students now?" (Lewis, 2002a, p. 4). Looking at the gaps between the real and the ideal, teachers work together to set a research theme that will inform the development of a research lesson. Multiple lesson study groups within a school may choose the same theme in order to work together towards the school's goals (Chichibu & Kihara, 2013; Lewis, 2002a).

Next, the group collaboratively plans a research lesson (Fernandez & Yoshida, 2004) or modifies an existing lesson to work as a research lesson. After identifying a particular lesson as part of a unit and subject, the lesson study group studies curriculum materials for knowledge about the content area and to find the best available lessons on the topic (Lewis, 2002a,). In this process, called *kyouzai kenkyuu* (study of teaching materials), teachers look for the key mathematical ideas they want their students to develop, as well as some of the challenges they expect them to face in learning them (Takahashi, Lewis, & Perry, 2013). This allows teachers to learn the subject matter deeply, anticipate student approaches to the problem and plan their own instructional activities in response (Shimizu, 1999).

The group will usually meet once a week for a few hours to plan the lesson. The group may take up to six meetings to make a detailed lesson plan that includes the steps of the activity, expected student responses, the planned teacher responses, and assessment notes (Fernandez & Yoshida, 2004). The group will also plan for how data will be collected during the research lesson itself, developing the lesson plan as an observational tool (Fernandez, Cannon, & Chokshi, 2003).

Generally, groups modify existing lessons rather than write lessons from scratch (Lewis, 2002a). This is done so that the group can benefit from the expertise of other teachers, read outside research on the subject and can dedicate their energy to planning for their specific classrooms. The purpose of lesson study is to increase student learning, not to create new curriculum (Chokshi & Fernandez, 2004; Lewis, 2002a). A Japanese teacher interviewed by Lewis and Tsuchida (1998) put it this way:

> If you shoot for originality too early in your development as a teacher, you're likely to fail. Initially, you must take a lot from others. But ultimately, to move to higher level of teaching, your lesson must become your own original thing, not simply an imitation of others. But it's through imitating others' lessons that you create your own authentic way of teaching.

More important than creating new lessons is developing "the eyes to see children (kodomo wo miru me)" (Lewis, 2002b), an ability to find evidence of student learning, motivation and behavior which Japanese teachers say comes as a result of participating in lesson study.

After the research lesson is planned by a group of teachers, it is taught by one teacher, while being observed by other teachers. The class is recorded in different ways (video, audio, notes and/or checklists of observations) and is discussed afterwards (Lewis, 2002b). Observing members of the lesson study group are responsible for data collection (Lewis & Perry, 2014). The focus of observation is the lesson, not the teacher. This is very different than than the way observation generally works in the United States, where classroom observation is generally of the teacher's behavior (Lewis, 2002b). The lesson that was written by the group is now used as a means to collect information about student learning and engagement. During the lesson, observers collect "evidence on students' engagement, persistence, emotional reactions, quality of discussion within small-groups, *tsubuyaki* (under-breath exclamations), inclusion of groupmates, degree of interest in the task" (Lewis, 2002b), etc.

After the lesson, there is generally a group discussion immediately following the class among the presenting teacher and the teachers who planned the lesson. A careful protocol is followed in order to help the presenting teacher feel comfortable and keep the discussion focused. The presenting teacher speaks first, giving an assessment of the lesson. All group members take responsibility for what happened during the lesson, since the lesson was created by the group. The group talks about their intentions in planning, differences between what happened and what they expected, and ask for feedback on particular aspects of the class. Discussion is focused on data that was observed in the class. Again, this data is generally about student response to the lesson (Lewis, 2002a).

The next step for many lesson study groups would be to come together later for a previously scheduled meeting in which the group would rewrite the lesson plan based on the

collected information and discussion from the debriefing session. A second teacher would then prepare to teach the revised lesson in a second class of students. The teaching/observation, debriefing, lesson revising cycle might continue, but generally the lesson is not taught a third time (Fernandez & Yoshida, 2004). In some schools, an observation may be become part of a public research lesson in which staff from the entire school, and possibly teachers from other schools, are invited to observe and participate in a debriefing discussion afterwards (Lewis & Tsuchida, 1998).

In order to support lesson study, most Japanese schools invite a "knowledgeable outsider" to talk with teachers in the lesson study group and participate in debriefing (Takahashi, 2014). The outside specialist has expertise in instruction. For a mathematics lesson study, the outside specialist would be someone known for his/her expertise in teaching math (Lewis, 2002a). This specialist may help a lesson study group start the planning process and then return to observe a public lesson and help school staff debrief the lesson. He/she may connect the research lesson to the unit and subject of which it is a part, and will also comment on parts of the lesson in order to help the group deepen their knowledge of instruction. These outside specialists will generally review the lesson plan in advance, prepare comments, observe the lesson, and make comments on specific details of the lesson (Takahashi, 2014; Burghes & Robinson, 2012).

Benefits of Lesson Study

The goal of lesson study is not the kind of overnight educational reform so often expected in American education, but a long-term incremental improvement based on local knowledge from teachers working with their peers to support their students' learning. The cumulative effect of this knowledge production has created an engine for improving Japanese education, with a

collective understanding that educational changes take time. The national Course of Study, which specifies basic learning objectives in Japan, is changed not more than every ten years (Lewis & Takahashi, 2013).

Though the process is slow, there are compelling benefits to lesson study as a primary means of professional development for teachers. Interviews and studies with teachers "indicate seven key pathways to improvement that underlie successful lesson study: increased knowledge of subject matter, increased knowledge of instruction, increased ability to observe students, stronger collegial networks, stronger connection of daily practice to long-term goals, stronger motivation and sense of efficacy, and improved quality of available lesson plans" (Lewis, Perry, & Hurd, 2004).

The process of lesson study allows teachers to think deeply about the content matter. In the same way that Japanese students are expected to learn math through problem-solving, instead of learning procedures before encountering problems, teachers learn content knowledge through the process of lesson study (Hart, Alston, & Murata, 2011; Lewis, 2009). A Japanese teacher: "Research lessons are very meaningful for teachers, because… they think hard and in a fundamental way about several critical issues, for example, "What is the basic goal of this lesson in this textbook?' 'How does this particular lesson relate to my students' learning and progress in this school year?' 'How does this lesson relate to other curriculum areas?'" (Lewis, 2002b).

One mechanism of lesson study is detailed and specific feedback about what happened in the classroom, as in a case of a special education expert observing a teacher with students who speak English as a second language. The observer was able to help the teacher understand that

speaking in paragraphs instead of sentences might cause difficulties for her students. The teacher was grateful for the feedback, not having recognized this possibility previously (Kotelawala, 2012). Being observed by other teachers and hearing evidence from the classroom allows teachers to see their teaching through others' eyes (Lewis & Tsuchida, 1998). Research lessons provide a mirror to one's practice (Lewis, 2002a).

> While tests and student work may offer information on what to improve, lesson study also sheds light on *how* to improve… Feedback from lesson study is immediate, specific to the school's curriculum and goals, and based on actual observation of the lesson. It comes from colleagues likely to have intimate knowledge of the students and their context, the people 'best positioned to understand the problems that students face and to generate possible solutions' (Lewis, 2002a).

Lesson study also allows teachers to build a shared vision of what good teaching looks like (Stepanek, Appel, Leong, Mangan, & Mitchell, 2006). In a methods course lesson study, an in-service teacher saw the effectiveness of a non-standard algebraic problem by seeing the responses of students (Kotelawala, 2012). In another lesson study, a Japanese teacher, reflecting on the effect of observing research lessons said:

> When I was young, I thought teaching was to make a point and explain students so that they can understand better. So, to me, back then, it was critical to find the 'technique' to do that effectively. After seeing the investigative open-ended lesson,

> I have come to think that learning is not what I had thought (Murata & Takahashi, 2002).

Working closely with other teachers is one of the most important features of lesson study (Fernandez & Yoshida, 2004). In the United States, teaching is treated as a solitary endeavor, with teachers succeeding or failing on their own. Lesson study allows educators to move towards a view of teaching as a professional activity open to collaborative observations, study and improvement (Stigler & Hiebert, 1999, p.xv). The main communication in this kind of professional development is between teachers, rather than between trainer and teachers, as in most American professional development (Lewis, 2002a). As one American educator put it, "Lesson study allows me to build relationships with my colleagues. [It] opens the classroom doors and minimizes isolation" (Lewis, 2002a).

In Japan, public research lessons have been used to disseminate information about teaching strategies or changes to national curriculum (Lewis & Tsuchida, 1998). Because the communication is teacher-to-teacher, the process is more efficient and effective than top-down professional development. Teachers pick up good ideas from other teachers.

> No one requires Japanese teachers to adopt the research lessons they see. But good new approaches tend to spread quickly. During a research lesson, if you see students gasp in amazement when they measure circles of various sizes and discover that the circumference is always about three times as long as the diameter, you will want to give your own students the thrill of discovering pi in this way (Lewis, 2002a).

Teachers who participate in lesson study tend to adopt a view of teaching as a knowledge-based profession that can be improved over time through careful and systematic study (Stigler & Hiebert, 1999). The intensive focus on the craft of teaching is undertaken with the assumption that teaching is a professional occupation, with the respect, responsibility and resources appropriate to its position. Lesson study gives teachers the primary responsibility for improving classroom practice. This emphasis on local knowledge driven by participants in the group provides a way to bridge the gap between research and practice (1999). Teachers use research from outside their classroom in the planning phase, but make final decisions based on their intimate knowledge of their classrooms. Through lesson study, practice becomes research (Lewis, 2002a). The goal is to restructure schools as places where teachers can produce knowledge and learn (Stigler & Hiebert, 1999).

Currently, lesson study is being used in K-12 schools in the United States to give teachers opportunities to strategize how to incorporate the Mathematical Practices that are part of the CCSS (Takahashi, Lewis & Perry, 2013). There is reason to believe that a similar process in adult basic education could help teachers make sense of the new HSE exams and strategize ways to improve instruction in ways that align with the CCRS.

Lesson Study in the United States

In 1993, Catherine Lewis started observing research lessons in Japan while researching a book on Japanese education. The teaching Lewis observed was part of a shift from "teaching as telling" to "teaching for understanding" that was actually initiated by education reform documents in the United States (Lewis, 2002). In 1998, Lynn Liptak, principal of Patterson School #2 in Patterson, New Jersey was introduced to lesson study by Patsy Wang-Iverson.

Patterson School #2 started a lesson study group in connection with the Greenwich Japanese School in Greenwich, Connecticut (Wang-Iverson & Yoshida, 2005). As part of the collaboration, teachers from Patterson visited Greenwich to participate in lesson study and learn from the Japanese teachers there (Stepanek, et al, 2006). The Patterson group was supported by American leaders of lesson study (Lewis, 2002a). Between 1993 and 2011, over 400 schools in the United States, primarily K-12, used lesson study for professional development (Hart, Alston, & Murata, 2011).

Lesson Study in Adult Education

Though there is a wealth of information available on the benefits, challenges, and best practices associated with lesson study (Lewis & Perry, 2008; Lewis, Perry, Friedkin, & Roth, 2012, Stigler & Hiebert 1999), there is a lack of documentation on lesson study groups with teachers of adult basic education. From experience and conversations with other teachers, I know that these groups have existed from time to time. Denise Deagan, Charlie Brover and Solange Farina ran a lesson study group at the Borough of Manhattan Community College (BMCC)'s Adult Learning Center in the 2000s (D. Deagan & C. Brover, personal communication, 2014). Mark Trushkowsky ran a lesson study group with teachers from young adult literacy programs in the spring of 2014 (Trushkowsky, 2014). Mark, Solange and I ran a lesson study group at BMCC from June 2015 until August 2016, and gave a presentation on the experience at the National Council of Teachers of Mathematics (NCTM) Regional conference with Tyler Holzer, one of the participating teachers (NCTM 2016 Regional Conference Program, 2017). To date, there has been no published research on the use of lesson study with teachers in adult basic education.

There are real challenges to implementing lesson study in adult literacy, since most teachers are part-time and are generally not paid for staff development time. Lesson study has also been difficult in the past because there was no standard curriculum in which to situate the project. Each teacher tends to make her own decision about what content to teach, what materials to use and how to order instruction. This creates a challenge when teachers plan instruction together for observation in different classes. If the challenges could be overcome, a lesson study group could be a way for teachers to improve their practice and refocus instruction on teaching for understanding.

Theoretical Framework

Constructivism

My theoretical approach to teaching and learning, whether it is with students learning mathematics or teachers learning new teaching strategies, is based on a constructivist orientation (Merriam, et al, 2007). We learn from our experiences and construct meaning from active interactions with the world. A classroom or professional development meeting is a structured way to learn from experience. People learn best when they are engaged in active inquiry while in dialogue with their peers and teachers, who act as facilitators or guides. A social constructivist approach recognizes that people learn in groups through discussion and consideration of problems they face in common (Driver, Asoko, Leach, Mortimer, & Scott, 1994, cited in Merriam, et al, 2007). The instruction in this project was informed by a problem-posing approach to math instruction, using open-ended, non-routine problems along with group work, strategies for productive struggle (Trushkowsky, 2016) and students' thinking at the center (Takahashi, 2006). This approach is based on use of the zone of proximal development

(Vygotsky, 1978), the understanding that students working in groups with a teacher's support are able to solve more difficult problems and learn more advanced mathematics than they would on their own without support. Integral to this theory is the idea that what students can do independently now is not an adequate predictor of what they can do with support and what they will be able to do independently in the future.

Situated Cognition

Situated cognition (Brown, Collins, & Duguid, 1989; Merriam, et al, 2007) is an appropriate theory for looking at the application of lesson study to adult education. Situated cognition asserts that learning is not in the learner alone, but in the relationship between the learner and the social environment (Merriam, et al, 2007). As Lave (1991) points out, communities of practice based on apprenticeship and mastery have been difficult to find in the American education system. Since learning is situated in complex, social activities (Greeno, 1997, cited in Merriam, et al, 2007), teaching should contain elements of real-world problem solving, including "ill-structured complex goals, an opportunity for the detection of relevant versus irrelevant information, active/generative engagement in finding and defining problems as well as in solving them…, and an opportunity to engage in collaborative interpersonal activities" (Young, 1993, cited in Merriam, et al, 2007). Though our classrooms are not usually structured in ways for students to experience legitimate peripheral participation in communities of practice, apprenticeship being an example, teachers do enter communities of practice and construct their identities as teachers as they develop teaching skills (1991).

Much discussion of teaching and learning deals with abstract principles for instruction. However, effective teaching (and discussion of teaching) requires staying as close as possible to

the instruction and the context, in the same way that textual interpretation in literature requires close reading. Abstractions can falsify the text (or experience). Even the idea that we can talk about "adult learning" (Merriam, et al, 2007) is the result of abstractions that blur distinctions between adults of different ages, cultural contexts, life experience, as well as the subject matter they are learning. Even if we can say that some things are true about all adults, these ideas are not especially useful when we are teaching in a specific context and situation. For example, we can say that adults have life experience that should be used in the classroom (Merriam, et al, 2007). How then is that put into practice in a specific class in Baltimore, with women in prison, studying college writing through journalism, in 1 1/2 hours a week, in the role of a tutor supporting a classroom teacher, on specific assignments that are due the next day (a teaching experience I had in 2014)? In the end, the abstractions probably aren't going to help a lot. Reflection on the specific situation will probably be most helpful. In order to understand the teaching and learning in our unique classrooms, we need to look closely at the particular complexity of that social space (Merriam, et al, 2007).

In considering improvement of teaching, the teacher is the learner. We seek increased knowledge of teaching, but this knowledge is not separate from the specific context of our classroom, our students and the material we teach. The greater understanding we seek is situated in that specific context (Lave, 1991). Just as our students learn best through authentic problem-solving experiences connected to real situations (Brown, Collins, & Duguid, 1989), learning to teach benefits from responding to the context of the class, rather than dealing with generalities across classes and content areas. Lesson study is a process that allows teachers to

consider their own classrooms, with the additional benefits of making use of collective problem solving (1989).

Communities of Practice

There is a history of adult education teachers using problem-posing, inquiry-based instruction in teaching math. In New York City, math teachers' circles have worked to improve the teaching of math through problem solving in adult basic education (Brover, Deagan and Farina, 2000; Appleton, et al, 2017). As math teachers in adult literacy, we also want to create communities of practice among our students (Merriam, et al, 2007). With students, we pose problems, encourage productive struggle (Trushkowsky, 2016), and try to "[provide] just-in-time assistance to enable confident action in situations where confident competence is lacking" (Fenwick, 2003, cited in Merriam, et al, 2007). With teachers, we work on math problems together in order to experience the struggles our students face, then we discuss applications to our teaching (Appleton, et al, 2017; Lewis, 2011). Lesson study is an example of a focused community of practice (Chichibu & Kihara, 2013; Lave, 1991) focused on a specific teaching challenge and set of classrooms.

Critical Theory

The final piece of the theoretical framework of this study draws from critical theory, since this project will implicitly and explicitly critique the status quo in adult basic education (Merriam & Simpson, 2000). Implicitly, as an alternative vision of professional development that requires collective learning and decision-making, it stands in contrast to current reality. Explicitly, the voices of teachers in adult education critique the teaching conditions in adult

education programs, the unreasonable expectations of the HSE exams and the damaging influence of bureaucratic requirements (Redmond, 2014).

Some basic values are at the heart of this research project and are in conflict with the dominant paradigm that governs how adult basic education is funded, supported and evaluated. Teachers should have full-time employment with living wage and benefits. Instead of funding adult learning centers through short-term grants, which creates instability, devalues expertise and forces competition over scraps, these programs should be have stable funding sufficient to employ staff full-time and offer a range of supportive services to adult students. Teachers should be respected as professionals, along with the expectation that they will work for continual improvement. For professional development, programs should be allowed to organize their professional learning activities, including lesson study. Best practices in curricula, teaching strategies and assessment should be informed by the work of teachers, out of their expertise and research in their classes. Professional networks and publications provide a means to disseminate good ideas. Those materials and strategies that are successful will be shared.

Clearly, this vision is very different from the current reality in which adult basic education teachers work part-time, are poorly paid and are not professionally respected. Programs chase grant money and can't afford to pay a living wage. The only professional development that counts for reporting purposes is top-down and sporadically offered. There is no mechanism for building on and drawing from teachers' expertise. In this way, this call for lesson study implicitly contains a critique of the current paradigm under which adult education is conceived. My hope is that this project can provide an element of a larger vision of what adult

basic education could be, “awakening people to the critical sense of what is possible” (Merriam & Simpson, 2000, p. 133).

Methodology

Lesson Study as Action Research

> *“The action research process is one of analyzing, getting facts, identifying the problem, planning and taking action on the problem, then repeating the cycle as new concepts and information result from the process.”* (Merriam & Simpson, 2000, p. 125)

Lesson study, as a form of practitioner research done in the classroom, is action research. The methodology is aligned with action research since it helps teachers improve their knowledge of teaching through group investigation and application of findings (Lewis, Perry & Friedkin, 2009). In contrast to traditional research, the findings of action research are applied immediately, in this case in the classroom (Merriam & Simpson, 2000). The goal is to use the results of research to immediately improve practice. Though the design of action research is sometimes determined in the process of research rather than at the outset, the lesson study research cycle is determined in advance, though the precise content and questions the group will investigate are not. Since participating teachers in the lesson study group became co-investigators in a collaborative project, we left the process open to changes in the course of the group’s work. As the research happened in the classrooms of the participating teachers, systematic sampling or selecting of participants was not possible. We cannot assume that the results of the lesson study group’s research apply generally since the participants were not sampled systematically (2000).

Lesson Study Research Design

For this research project, I planned and implemented a mathematics lesson study research cycle with my colleague, Mark Trushkowsky, the math professional development coordinator for the CUNY Adult Literacy/HSE Program. Mark took the role of leader of the lesson study, which involved facilitating meetings, introducing lesson study as a practice, bringing materials, suggesting lesson topics, and helping the group make decisions. He also maintained a web site for the teachers which included our schedule, weekly notes, lesson plan drafts, readings and resources. Along with my role as researcher, I participated as a teacher in the group. During the fall semester of 2017, I taught a weekly HSE math class so that I could participate actively in the lesson planning process and potentially teach the research lesson.

The instructional content of this lesson study was based on the math section of the CUNY HSE Framework (Brandt, et al, 2015). The CUNY Framework focuses on functions and algebra, so our expectation was that our research lesson would target an element of these topics that is missing from the curriculum or has been particularly difficult to teach.

In preparation for the lesson study, Mark Trushkowsky and I completed the following:

- Wrote draft semester syllabus and lesson sequence for using the CUNY Framework in an HSE class.
- Chose readings to introduce Japanese lesson study to the group of teachers, including A Lesson Is Like a Swiftly Flowing River (Lewis & Tsuchida, 1998) and Aspects of Mathematics Teacher Education in Japan: Focusing on Teachers' Roles (Shimizu, 1999).

- Chose content resources beyond the CUNY Framework, including Chapter 3 (Patterns and Functions) from A Collection of Math Lessons (Burns and Humphreys, 1990) and About Teaching Mathematics: A K-8 Resource, by (Burns, 2007).
- Made a schedule of the lesson study cycle to share with teachers and their program manager.

The lesson study schedule consisted of seven planning meetings, two class observations, and two post-observation meetings, which happened approximately once per week in the fall of 2017:

- Introduced Lesson Study and developed research theme - 1 meeting
- Researched subject area and chose content goal - 1 meeting
- Planned research lesson - 3 meetings
- Conducted research lesson with observers/Discussed research lesson with lesson study team and observers - 2 hour class, followed by 1 hour lunch meeting
- Rewrote lesson - 1 meeting
- Conducted research lesson with observers/Discussed research lesson with lesson study team and observers - 2 hour class, followed by 1 hour lunch meeting
- Reflected on lesson study - 1 meeting

There is an expectation in the participating adult learning center that teachers are paid for participating in professional development, since they attend these sessions outside of their normal work hours. As is standard procedure, participating teachers were paid a stipend of

$30/hr and received professional development credit from the New York State Education Department for being part of the lesson study.

Case Study

I followed a case study model to capture and analyze participating teachers' experience in the lesson study research cycle. I have given the teachers and the adult learning center pseudonyms to protect the teachers' privacy as subjects of research. After inviting math teachers from the city broadly, we selected four teachers from the same adult learning center, referred to herein as NYCALC. The lesson study planning meetings and class observations took place at the adult learning center. The debriefing meetings after each of the two research lessons took place at a restaurant nearby. The center is well-loved by the teachers, one of whom said, "[NYCALC] was the first place I ever taught.... It's always great to be there. I really like that place and what people do there and how people work together. I really enjoy being there" (Kimia).

We were to lucky to pull together an experienced and talented group of teachers for the lesson study project. Diane has taught math in adult education for more than 10 years, loves math and maintains an extensive library of math books. Ben coordinates the high school equivalency program, has been a New York State adult education teacher leader, and has run mathematics meetings at NYCALC as well as city-wide workshops. He is also a member of a math teachers' circle. Victoria has taught in various adult education programs and tutors children in mathematics. She has participated in a range of professional development workshops since becoming a teacher. Kimia ran a college transition program, was part of a math teachers' circle and has done various types of professional development. The participating teachers each had

experience teaching different subjects in adult literacy and high school equivalency, but were all teaching mathematics at the time of the study.

At the beginning of the lesson study process, I conducted one-on-one interviews with participating teachers based on the questions in Appendix A. With the participants' permission, I also audio-recorded conversations during planning and post-observation meetings. I collected meeting notes, planning documents, lesson plans, and individual notes from research lesson observations. After each meeting, Mark shared a report out with participating teachers documenting the meeting with notes, photos, and decisions. In our final group meeting, I recorded a discussion of the question, What was the most useful or valuable aspect of our lesson study work together? At the end of the lesson study cycle, I transcribed participating teachers' initial pre-lesson study interviews and provided them individually to the teachers for their review, followed by a second interview based on the questions in Appendix B.

As a "new approach to ongoing problems" and "a particularly useful methodology for exploring an area of a field of practice not well researched or conceptualized" (Merriam, et al, 2007), case study is an appropriate approach to evaluate the effectiveness of lesson study in adult education. A case study allows for a thorough description of an experience of lesson study in the context of adult basic education. This kind of research has previously been done with K-12 teachers using lesson study (Lewis, Perry & Friedkin, 2009; Takahashi, Lewis, & Perry, 2013). At the conclusion of the lesson study cycle, I compiled the materials collected over the course of the research and looked for evidence that would help me understand the experience of participating teachers (Harel, 2008).

When I first started planning this research, I imagined that it would be an example of participatory action research (PAR), since the participating teachers become co-researchers (Merriam & Simpson, 2000). In the lesson study research cycle part of this project, these teachers conducted practitioner research in their classrooms. In this case study based on the lesson study, however, the participating teachers were subjects and I was the researcher. I kept the process of collecting and analyzing case study data open to teachers' participation, but their primary focus was implementing and understanding the research done with students. I expected that participating in an additional case study analysis of their own experience in the lesson study research cycle might have been distracting from their main goal. For that reason, I don't consider this project an example of PAR.

Findings

Beginning the Process

Our first meeting at NYCALC was in October of 2017. Mark and I set up a circle of school desks in an ESOL classroom crowded with extra tables and cabinets. The classroom is connected to a warren of offices, storage and a break room where teachers eat lunch and chat between classes. Photographs of teachers and students, posters, and poetry from the last 20 years line the walls in the hallways.

When everyone arrived, Mark started the meeting by reviewing the agenda and then led us into an icebreaker called the History of My Name, with each person telling stories about the origin of their first name, last name or nickname. Teachers were then given a chance to share questions about how the lesson study cycle would work. They had all read A Lesson is Like a

Swiftly Flowing River (Lewis & Tsuchida, 1998) in preparation for the meeting. The discussion focused on logistics such as the number and length of meetings, how research lessons work, and requirements for class coverage. The teachers then talked about our interest in lesson study. From this discussion and individual pre-lesson study interviews, I learned about the teachers' comfort teaching math for the TASC exam, their experience using the CUNY Framework, and their interest in lesson study.

Understanding and teaching TASC math. The teachers had different experiences understanding and teaching the mathematics content on the TASC exam, but there were commonalities. When the HSE test changed to the TASC, all of the teachers had to do work to become comfortable teaching it. Some teachers, including myself, had to learn the content themselves before thinking about how to teach it:

> Once the test changed, I really had to go back and study. I had been teaching for many years. This was material we didn't necessarily have to cover... I don't know if I ever truly had to deal with functions, so I think I had to learn them once the test was changing... (Diane)

> Some of it was what I remember from what I learned in school, but there were other things in terms in functions. Even that word. I don't remember that work being used. I remember having to go over it and think, what are functions and kind of refresh my memory because yes, we touched on it in high school, but we didn't call it that. And there were some things that I had to go over on my own and teach myself. (Victoria)

All the teachers had to think about how to teach content that wasn't included on the previous version of the GED. This involved helping beginning-level students make connections to the abstract style and the formal concepts of the TASC exam.

> I have taught an advanced class for last two years... I feel that I'm comfortable with functions and algebra. I spent a lot of time reading practice questions and trying to use that as a basis for some of my class…. Before I taught the advanced math class, I taught a beginning math class right around the changeover from the GED to the TASC. I felt that helped me as well because I was working with beginning level students and I had to think how this test is a very difficult test, how do I work with students at a very beginning level to expose them to some of the types of language that appears on the test, but also some of the concepts of algebra. (Ben)

The CUNY HSE Math Curriculum Framework. All of the participating teachers were familiar with the CUNY Framework. Ben and Kimia had taught all of the lessons at least once. Victoria found the lessons on teaching algebra with patterns really useful. Diane had taught materials from the first unit on introducing functions, but hadn't taught the other lessons. Overall, their experiences teaching with the Framework were extremely positive.

> Teaching the Framework was really helpful.... I guess the main [way I learned new HSE content] was teaching the Framework. That was the refresher for me. (Kimia)

> I've used the Framework before. I feel that does a very good job of introducing functions in a way that makes sense for students. It's very planned out, lesson by lesson, introducing a little more about functions each time. (Ben)

Victoria's positive experience teaching with the curriculum was consistent with the other teachers:

> I feel really good now teaching it. The [CUNY Framework] opened up a different idea in terms of the patterns. The idea of using patterns to teach algebra was different for me…. And it really worked for my class. I like that idea of patterns. You can actually get an equation looking at patterns. (Victoria)

Challenges in their work. Though I didn't ask specifically about challenges in my initial interview with teachers, the topic came up naturally. Some teachers talked about the challenge of an exam with more difficult, abstract content. Others spoke about how there was a lack of adequate curriculum in some topics, such as geometry. A couple of teachers agreed that geometry would be a useful topic on which to focus our work. Others spoke about the challenge of balancing administrative work against the work necessary to improve teaching in the program. Finally, one teacher spoke movingly about the frustration that she feels when she doesn't see her students making progress.

> It is new territory for the students and for ourselves... I know how to teach skills. I know how to do that, but two things are happening now. Just teaching skills is not enough and a lot of my students have never been successful with the skills and they still aren't... In previous classes, semesters, we've been able to move forward with topics and scaffold lessons, and that kind of thing. I'm just not finding that this time... I have some students who have been with me for three and four semesters who are still asking me questions about subtraction. That blows my mind. I feel like If I go any slower, I'll be moving backwards… I'm very frustrated. (Diane)

Interest in lesson study. Other than Mark and myself, none of the participating teachers had been part of a lesson study group before. Ben had been interested in lesson study since reading the Teaching Gap (Stigler & Hiebert, 1999), since he felt feedback from other teacher would improve his lessons and his teaching.

> The idea of teachers coming together is something that appeals to me because oftentimes, at least when I started out as a teacher, it can be a very isolating activity, when you're responsible for teaching a three-hour class. I like the idea that it's multiple teachers working through a lesson or a problem that multiple teachers could use in their classroom and you can then have a discussion about it afterwards, a discussion about what worked and what didn't work. Not only does it improve the lesson, it helps improve the teacher…. I felt this way the last math meeting we had, I did this lesson on pi and measuring objects and it was a really good lesson, so maybe I didn't get much feedback or much criticism... Sometimes,

> I think a teacher might not want to say something like that... You could say, hey, have you considered trying this? There are a couple teachers who do that, but not everyone does that. (Ben)

The other teachers were unfamiliar with lesson study before joining the group, but after learning about it they had specific ideas about how it might help their teaching. The teachers were interested in new teaching methods and materials. One of their primary interests was the opportunity to collaborate with other teachers. In conversations with teachers, I learned that there was a hunger to work together, to share materials, to get feedback and to help improve their teaching. Kimia, for example, said, "I wish we met more as math teachers and I wish we did a lot more work together."

She continued by talking excitedly about the opportunity to spend time analyzing a specific piece of mathematics content: "It seems like this is an incredible opportunity to actually spend time looking at one thing in depth, which we don't often do... You're teaching one thing and then the next thing and the next... so I appreciate that about it" (Kimia).

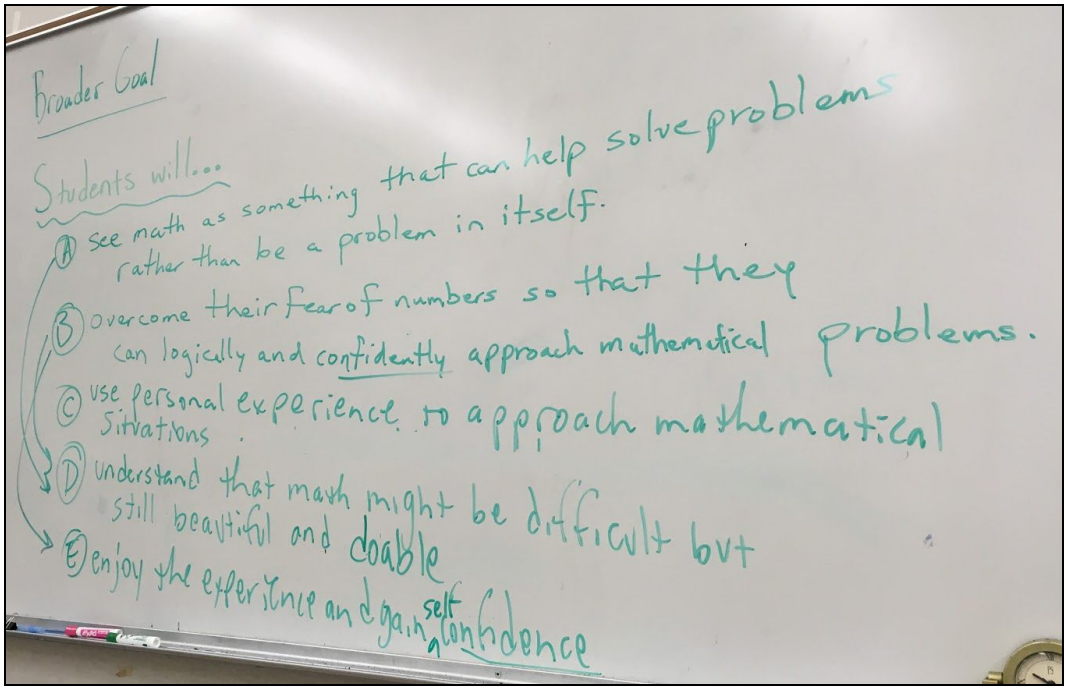
Victoria was really interested in how lesson study could be used in her daughter's elementary school to improve the mathematics education she was getting. She wanted to experience the lesson study so that she could then go to the school and advocate for the implementation of this practice. From reading about lesson study, she found the idea of teaching through problem-solving compelling as an approach for her daughter and other kids of her age.

The NYCALC Lesson Study Cycle

After our discussion of our interest in lesson study in the first meeting, Mark moved us into a brainstorming activity to define the goal or theme of our research lesson. We started by writing on two questions: 1) Thinking about the students you serve, what qualities would you like them to have when they leave NYCALC? 2) Where are they at right now? We each wrote individually and then shared our writing. In small groups, Mark asked us to discuss our answers to the first two questions and then consider a third question: What are the gaps between where students are now and where we want them to be? In my small group, Victoria talked about how she wanted her students to think logically and be able to make sense of information. Kimia's ideal was that students would feel that "math might be difficult but that it can be beautiful, stimulating and wondrous." She also wanted students to feel that math is "difficult, but not undoable." In conversation with the other small group, we developed the following two analogies to guide our thinking:

- *A frightened person is not a thinking person.* We wanted to help our students develop enough confidence to deal with ambiguity and use their background knowledge to make sense of mathematical situations.
- *A comfortable cook does not need a recipe.* We wanted our students to understand rather than memorize. If they understood situations, they could recreate formulas and procedures. If they didn't understand, they would always need to blindly follow procedures.

Mark took notes on our ideas and helped us start to pull them together into a unified vision.



Through small group discussion, negotiation and multiple drafts as a group in the next meeting, we developed the following two-part goal for our research lesson.

> Students will enjoy math as a conquerable challenge by building on successes and developing a systematic approach to problem-solving.
>
> *We want students to see math as a challenge, not as a problem. They will have the confidence to not view struggle as a negative statement about their own ability, but as an expected part of learning. They will use their personal experience and the information given to approach mathematical situations with the spirit of a scientist, with the confidence to move forward and try things that may not work. With each challenge they work through, they will gain confidence in their ability to use a systematic approach as they make sense of unfamiliar situations or problems.*

This initial discussion of our lesson study's goals relates to the research question: How does lesson study affect adult education teachers' beliefs about students' abilities? The lesson

study process forced us to think about our students as whole people, with math learning as part of general development. This is different from normal discussion of math instruction that focuses on content or pedagogy, but generally does not include students' affect.

Based on individual conversations and discussion in the first meeting, we decided that the mathematics content goal for the research lesson would be based on the following two geometry content standards: 1) Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. 2) Explain volume formulas and use them to solve problems (Pimentel, 2013). Our goal was to connect these geometry standards to algebra and functions concepts in the CUNY Framework.

Over the course of the next four meetings, the group reviewed a set of possible lessons and wrote a research lesson based on the lesson, Evaluating Statements About Enlargements from the Mathematics Assessment Project (MARS, 2015). In the lesson, students would be asked to consider different figures (rectangles, circles, rectangular prisms, spheres, cylinders, and cones) and would then work in groups to evaluate statements related to the measurements of these figures (the area of a circle, the volume of a cylinder, the surface area of a sphere, etc.). Each statement makes a claim about what happens if one, two or three dimensions of the figure (length, width, height, radius, etc.) are doubled in length. Students would be asked to decide if the statement is true or false and offer a convincing explanation of their answer. Here are a couple examples of statements students would have to evaluate:

- If you double just the width and width of a rectangular prism then you double its volume.

- If you double just the height of a cylinder then you double its volume.

Ben taught the first version on November 29th, 2017 while the rest of us observed. We were excited to hear echoes of our research lesson goal when a student who had initially been overwhelmed by the task later reflected: “We should do this kind of activity more often. It challenged me. It was more challenging, but good challenging.”

The lesson study group ate lunch together afterwards at a Dominican restaurant in the neighborhood and shared notes on the research lesson. In the following meeting, we revised the lesson, which Kimia then taught on December 13th, 2017. That class was also followed by a debriefing conversation over lunch. In our last meeting on December 20th, teachers reflected on the lesson study experience and discussed possible next steps.

Lessons Learned

From analyzing interview and meeting transcripts, the following themes emerged as takeaways from the lesson study cycle.

Planning for Instruction. All of the teachers spoke of the benefits of planning for instruction through the process of writing and revising a lesson plan together with other teachers. They talked about the value of putting together a thoughtful and meaningful lesson, as part of a series of classes in a semester, each sequentially working to build students’ understanding. For some, lesson study allowed them time to practice the technical skill of building the pieces of a lesson plan for the first time.

> I may have expressed this before, that being in adult ed for many years, we weren't really required to do formal kinds of lesson planning or anything. It was

> just, get in there and teach these people some stuff. And so I never felt competent or comfortable planning lessons in a formal way, so the lesson study gave me a way to feel more comfortable and understand planning a lesson…. Now I'm thinking about how to set a goal, how to plan towards a goal, to account for what needs to happen, to anticipate and plan for the unexpected and to know how and if the lesson goal has been achieved…. I'm pretty prepared for classes, but with the lesson study, it was even more. Writing the stuff on paper, just the timing was so tight, so it tightened some of the screws in my own teaching. (Diane)

For Diane, participating in the lesson study helped her practice the technical work of preparing to teach. It is clear that her identity as a competent professional has developed with her participation. Also implicit in her response is the idea that she will be better prepared for evaluation of her work. Diane also spoke about how the lesson writing process helped her recognize what work she could do to improve her teaching. It helped her see a path towards a sense of efficacy in the classroom (Lewis, Perry, & Hurd, 2004).

> That I need to do more, that I need to take on the responsibility, take more of the responsibility for my students' success and try again... It means being more thorough, planning the lessons, giving more thought to what I want students to take away from a lesson, to actually have a real lesson plan to see it through, to talk less and give them more space, and hope that the learning happens or find measurable ways to see that learning has happened. (Diane)

Teachers also spoke about using the lesson writing process as a way to imagine themselves in the classroom and to think through the conversations they would have with students.

> I realize maybe I need to have things a little more outlined for myself… maybe I should have sat down and done a trial run for myself, like almost rehearsing a play, see what makes sense. Sometimes, when I see something on paper, I'm like, Oh, I got this. I understand this. I just have to do A, B and C and it will be fine and maybe I should have done a walkthrough a little more…. I guess that rehearsing that ahead of time, maybe doing it out loud and then seeing how it might connect, sometimes when I see things on paper, it seems to make perfect sense, or it seems like mentally I have it, but not until I do it do I really feel what needs to be changed. (Ben)

Ben went on to talk about how he liked that he was able to teach the research lesson first, when it was still in an earlier draft. It took some pressure off and allowed him to think of it as a rehearsal that would help all of us understand what could be improved.

Kimia explained a realization she had about the amount of time she spent talking in class, though it wasn't the first time she had thought about it. In fact, in our first meeting she talked about how she knew her lesson that morning had been unsuccessful because she found herself talking too much. Participating in lesson study helped her realize how this connected to planning instruction.

> I try to overcompensate by talking if I haven't planned well enough or scaffolded well enough and they don't get what I'm trying to get across and so I spend a lot of time trying to explain it…. I think I realized that the energy that I spend should not be in the class. The energy that I spend should be outside of class. The preparation should be where the energy goes, of course knowing what the steps are,... but the prep is the bulk of the work, and when you're in the classroom, the students are the ones who are supposed to be doing the work and you're just observing what you planned to do and they're the ones talking and processing and trying to figure things out. It's like being a midwife, you're not doing the work, somebody else is doing it, you're just there to help. (Kimia)

Overall, it seems clear that the focus on planning for instruction helped these teachers increase their knowledge of instruction (Lewis, Perry, & Hurd, 2004).

Teaching and Observing the Research Lesson. Ben and Kimia volunteered to teach the research lesson in their classes. The teachers in our lesson study group decided not to invite outside observers. It can be nerve wracking to voluntarily invite even the five other teachers in a peer group to observe you teach and videotape the lesson, especially since we don't have a culture of collaborative observation in the United States. However, through planning the lesson together, these teachers knew that the goal of observation was to learn about the effectiveness of the lesson and how our students would respond. We wouldn't be evaluating Ben and Kimia as teachers.

> [Kimia] said to me that she was so glad that I went first, that she got to see me teach…. I think sometimes it even helps to see a teacher teach even if the lesson doesn't go that well or they make some mistakes, because it helps you observe. That's a good idea, maybe I'll use that and write that idea down. Oh, I didn't think to do that in the classroom, but also to see a teacher make a mistake, maybe [Kimia] saw something I did and said, I don't want to spend as much time on that. That took too long. And I didn't feel that it was judgmental at all. I think that was the most helpful aspect…. The goal was the lesson and I really felt that. (Ben)

Though the decision to teach the research lesson probably felt risky, it seems that there was an additional benefit for those teachers. What had been abstract in the planning became concrete when they had to plan for the next day's class. Suddenly, little details had to be resolved since they were going to walk into class the next day and teach the lesson.

> And I realize that once I taught it, that was also a different experience for me. I think if I hadn't taught it, my relationship to it would have been very different. I'm a very hands-on person, so for me to experience something, I have to do it in order to completely internalize it and I know I have to do it in order to know the depths of the challenges and the joys of it, the pitfalls and all that. When you're in the driver's seat, it's a whole different ballgame from when you're watching it. So I was happy to do that even it's really difficult to put yourself in that position, so I'm really glad that I did it. And I was really glad that [Ben] did it first and I was able to reflect on that. I think that was incredibly valuable. It was interesting how that happened as well, because I think that's the way he is, you know, let's just try

> this, and for me, I kinda want to know that it's going to work before I do it, so it's perfect that he did it first. I think both of these positions have their challenges in different ways, but they suited our approaches, the way we do things individually. (Kimia)

Collaboration vs. Isolation. A common theme in the conversations with the teachers was the natural isolation of teaching. The lesson study process, though time-consuming and often exhausting, was the opposite, in that the lesson planning and the teaching happened as a group. It had the effect of inviting other teachers into the presenting teachers' planning processes and professional performance.

> There are always a couple of things that might come up in any lesson that surprise me, something that I don't anticipate, but I think this lesson study, the process of talking it through with a group of 4 or 5 people really took some of the surprises away. There are times where I have to do stuff on the fly and I'm sure I did a little of that when I was teaching the lesson, there are things I assume that students know that they don't, but I think that the preparation ahead of time really helped prepare me for some of those variables. (Ben)

The lesson writing process was also a way for us all to come to a consensus on the content and form of the lesson. Over the course of a few meetings, in evaluating the lesson's statements ourselves, we looked for ways to understand the mathematics so that we would know the best way to teach it. The lesson planning process wove together discussion of mathematics

and teaching strategy. By working on the lesson together, we may have initially started out with different ideas, but came closer to consensus the more work we did.

The process of planning, observing and reflecting also created opportunities for us to learn dozens of little teaching strategies from each other. We learned the value of preparing mathematical statements on chart paper that could be placed quickly on the board, so that students wouldn't have to wait for us to write. We saw how you can make sure students are in their groups and ready before you give them their task. We saw the benefit of writing today's vocabulary on the board with the agenda so that we remember to review it with students.

In our conversations after the research lesson, we also had opportunities to learn things like how we each drew a rectangular prism. After observing Ben's class, we realized that not knowing how to draw the figure limited students' strategies for evaluating statements about the figure.

> Mark: Shabana struggled for 5 minutes trying to draw a rectangular prism. She had a rectangle...
>
> Diane: And this slanted line that came to a corner...
>
> Mark: I have a photograph.
>
> Kimia: That's a hard thing to know how to do. I remember as a kid somebody showing me how to do that and I thought, oh my god, this is amazing.

Eric: Nadia was practicing it, too. She sort of gave up on it and just started dealing with numbers, but she started by... copying this and she was drawing that diamond.

Kimia: Oh my god, I want to show them how to do it.

Mark: I know. Because they all abandoned drawing pictures or the ones that were struggling because they couldn't and it was too much, so they stopped. That was the only one they tried.

Kimia: Oh my god, I want to do that. I want to do that. I want to show them how to do it. Because there are different ways of doing it. There's two rectangles. Or there is just doing one, and doing that [demonstrates].

Victoria: That's how I do it.

Eric: Wait, wait. What do you do?

Kimia: I do this. You go like this [demonstrates]... I love it!

The original lesson that we adapted had various statements on cards for students to evaluate. Realizing that our students might not be able to get through all the statements in the class time, the group discussed the possibility of using only the cards related to a rectangular prism. In that conversation, Kimia's comments helped Victoria realize the benefit of modifying lessons and adapting them for our classrooms.

Well, I know there was one time when I was trying to think about the 12 cards that we had, and how to help the students to get through all of them, and I thought

> what if one group is working on two of them, or one group is working on one shape and another group is working on a different shape, and Kimia said, is the goal for them to get through all the cards or for them to really get something out of the lesson? I think that's how she phrased it, but it made me think, yeah that's true. Do we want them to go through all of the cards or really understand what they're working on?... My mind was doing the whole lesson, because that's what was there. That was the lesson, that's what we had to do. But I didn't think about cutting parts of the lesson out. I didn't see that as a possibility. (Victoria)

Teachers reported that getting together and thinking with each other opened their perspective and helped counteract isolation in their teaching.

> Being a teacher is isolating. Despite the fact that we work in a place in small quarters with other people, it doesn't necessarily mean that we are always talking about our teaching. I think at [NYCALC] because of the ways the rooms are set up, we eat lunch together and talk with each other, but it doesn't necessarily mean that we are always talking about what we do in the classroom. Sometimes, we do informally and sometimes we don't, but to have a formal setting where you're focused on one particular task puts your feet to the fire. You have to do it and there's no choice, so you're forced to talk about this. Forced in not a bad way, but I think it's always great to spend time processing things with other people who are thoughtful and reflective and are thinking about the same things from whatever

> perspective and I think you grow when you do that with other teachers. It's a great opportunity to do that with other people that are smart and thoughtful. (Kimia)

As a result of collaboration in the lesson study, our group developed a stronger collegial network (Lewis, Perry, & Hurd, 2004). Teachers worked together to plan instruction and made a commitment to work together in the future.

Learning in groups. For Victoria, one of the biggest takeaways was the benefit of having students work in groups. As she observed students in Ben and Kimia's classes, it stood out to her that students were supporting each other, filling in gaps, and that they really enjoyed working together.

> I think working together with someone else, working as a pair, having a partner to work with, was helpful to both students because they bounced ideas off each other. Maybe a student was thinking of doing it one way but another student was thinking of doing it another way. They were able to discuss that or compare it or see which one was better, and just work together to work through the questions, the cards. They prefer to work as a group. (Victoria)

It seemed that this made a strong impression on her. For her own experience using lessons where students worked actively in groups, she knew group learning based on exploration could be helpful for adult students.

> I would love to make every lesson like that, but it's not something I know how to do for all lessons that I teach, so it just made me think, what am I doing to make the lesson as understandable as possible to the students and I do believe that

> making it interactive is the best way. I've always believed that. The fact that they worked with someone, they interacted with another student. They worked together to, either by drawing it out, and geometry is a lot about drawing, so drawing the shapes out, labelling... It felt like it was more hands-on. The use of manipulatives. That's what I mean. Instead of just giving out a worksheet and having them work individually on questions, it was more interactive than working alone. I think it helps them think better. It helps them process information better. I think the information will stay with them longer when it's interactive than when it's not. They tend to forget more what they were taught. When it's more interactive, it's more enjoyable. I think you remember things that you enjoy, rather than things you find boring. (Victoria)

Her reflection on this experience shows the need for continued collaboration and support. When teachers are in the classroom every day, it's hard to design lessons that allow students to work in groups. It's a heavy load to put all the responsibility for creating and implementing engaging curriculum on a teacher by herself. However, through connections with other teachers, more interactive lessons can be created and shared.

> I think it would be good to work together with another teacher... because just doing it alone by myself, I can come up with some ideas, but when you're working together with someone else and coming up with an activity that would be interactive, I think that would help in making that happen. (Victoria)

Understanding Student Thinking

In my initial interviews with the teachers, I asked what they wanted to understand more about how their students learned functions and algebra. In their responses, they tended to speak more generally about how their students learned or what struggles they encountered in learning. This became an important topic we would return to in our reflection on the lesson study cycle. After observing students in the research lesson, we became curious about the specifics of what they understood and had confused in the lesson. It was an opportunity for us reconsider our beliefs about student abilities and to sharpen our vision in looking at student understanding.

Things we took for granted. In our lesson, a key skill necessary for students was the ability to look at formulas and understand how to substitute values in order to calculate a result. For example, the volume of a cylinder can be found with the formula, $V = \pi r^2 h$. I'm going to guess that the reader hasn't seen this formula for a while. Stop for a second and think about what each part of the formula means. How much of that formula does the average student entering high school equivalency classes understand?

Understanding $V = \pi r^2 h$ involves knowledge of mathematics and of the conventions of symbolic representation in mathematics. Here is an incomplete list of what students have to know in order to understand and use this formula successfully:

- V stands for volume, which is a 3-dimensional measurement that equates to the counting of how many cubic units (cubes) would fit in a particular figure.
- The = sign represents the equivalency of everything on the left side with everything on the right side. Students often misunderstand the = sign to mean simply the answer, similar to how the symbol functions on a calculator. For example, 3 + 5, hit equals and

the answer is 8. So, if they were to look at the equation, 5 + 2 = □ + 5, they might say □ should be 7 to make the equation true. This is a common misunderstanding that can interfere with proper use of formulas.

- π is a constant that represents the ratio of the circumference of a circle to its diameter. Commonly, people use 3.14 to represent pi, but since it is an irrational number, 3.14 is an approximation.
- The length across a circle and through the center is called the diameter. Half of this length, from the center of a circle to the outside circumference, is called the radius and is represented by the letter *r*.
- The little number 2 above the *r* is an exponent and means that the quantity for *r* should be multiplied by itself ($r \times r$).
- πr^2 by itself is a calculation of the area of a circle, in this case the circle at the top of the cylinder.
- *h* stands for the height of the cylinder.
- And then there is a crucial understanding that, in algebra, a letter (or symbol) next to another letter (or symbol) means that the two quantities represented should be multiplied. So, πr^2 means 3.14 multiplied by whatever $r \times r$ turns out to be.

In our lesson, students had to evaluate statements about how a measurement would change if a dimension in the figure is doubled. Considering a statement such as… *If you double*

just the height of a cylinder then you double its volume.…we expected that students would use the formula for the volume of a cylinder to determine if it was true.

In comparing observation notes over lunch, we noticed some of the things students misunderstood as they tried to use formulas to evaluate the statements (and the things that we had taken for granted). Since the observing teachers each moved around the room while students working in small groups, we had to piece together what happened.

> Mark: I was in two different situations where Camille corrected or helped her... and I happened to be standing right there...
>
> Kimia: Camille wasn't necessarily right in what she was showing her, either. Did you notice that?
>
> Mark: The two times I saw she was.
>
> Kimia: The few times I watched, she wasn't. Because what they were doing, they were adding the length, width and the height and then doubling it.
>
> Mark: Three groups did that! I thought that was so interesting. They looked at the l, w, h and they were adding them and then multiplying by 2.
>
> Eric: I'm confused. I thought that was attached to the simplification of the perimeter formula, but it was actually grabbing a wrong formula.
>
> Mark: No, that was a separate issue.
>
> Kimia: No, Camille had a prism and she had labeled it 4 by 4 by 7. And then she said 4 plus 4 is 8, plus 7 is 15, 15 times 2 is 30.

Eric: And that was the volume?

Victoria: That was card number three, where it says if you double the length and the width and the height, and they thought that doing that was doubling each one... Adding each one and then multiplying it by 2.

When we observed students working on the task, the teachers noticed that a number of students added quantities when the formula indicated that they should multiply. For example, the volume of a rectangular prism can be found with the formula $V=lwh$. A few students added the length, the width and the height. They clearly didn't understand the convention of indicating multiplication by placing variables next to each other.

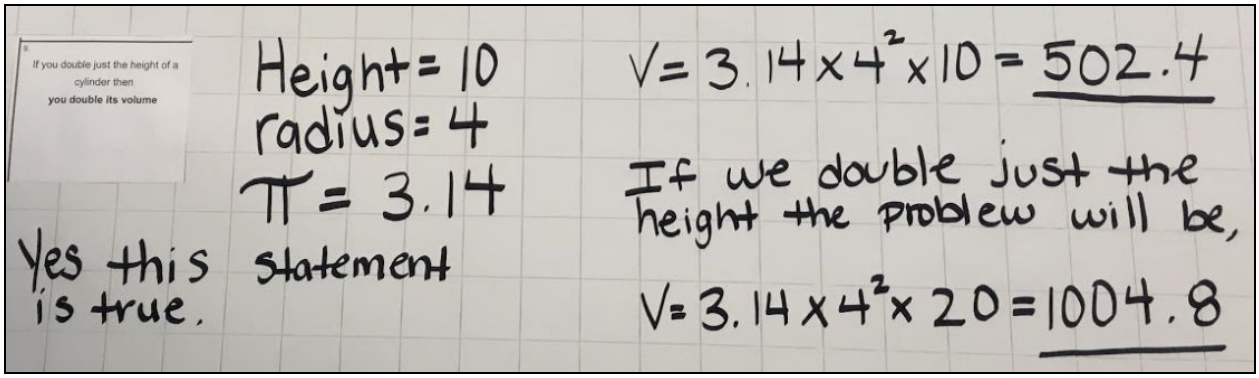
Other students were confused by the idea of doubling. Was it because they thought the exponent ^{2} in the formula meant doubling? If so, maybe something was already doubled in the formula, so there wasn't a need to do it again? Or was it that they didn't know what the English word *double* means? Many of the students in Ben's class were in a high-level ESOL class. Even native English speakers may have had an imprecise understanding of doubling, in that it implied that a quantity grew larger, but not in a specific way. In observing students in the task, we realized that students struggled with doubling:

In terms of the formulas that we worked on, that first question that you asked from the lesson study were thoughts of where student learning and understanding emerged, there were a couple things that I realized while we were doing the lesson, just things that we take for granted that students know. For example, the doubling issue. I never would have thought that student would struggle with that

concept, what the doubling means, to double a number or how to use a formula. That's something, I guess I look at the formula and to me it's self-explanatory, how to use it, but to our students, it wasn't self-explanatory. (Victoria)

Understanding where your students might have misconceptions is extremely important knowledge for teaching. I expect that our discovering of the confusion around doubling and notation of multiplication of variables will stay with teachers and prove of benefit the next time they are planning to teach students to use formulas.

There were also some students in the class who used the formula correctly, substituting lengths for the radius and height, doubling just the height, determining the volume of the two cylinders and correctly deciding that the volume had doubled.



How do adults learn? An important theme that emerged in our conversations was the challenge adult students face in learning the wide range and depth of content needed to pass the HSE exams. Adults have jobs, take care of children and parents, worry about money, not to mention in some cases having other concerns related to poverty, poor health and housing insecurity. A question arises of what success looks like when working with adults, especially at the more beginning levels.

> This is always the difficulty with teaching adults because they have a life. They don't necessarily have time, but just to get into a study habit. I open these questions, how many of you went on a web site and studied? Did you put in some hours? The number of people who do are minimal. Did you do your homework, did you work on this? I got home and I didn't know what to do. I got confused as I soon as I walked out of the classroom. How do we rectify this? How do we make it stick? What is knowing? Is it a momentary knowing? I get it, this is nice, and then the following week, it's just gone. How fleeting is the knowing? How can we rectify that? (Diane)

After teaching the research lesson, Ben returned to the content area with students a few weeks later. Even the details of the research lesson which we planned so carefully for months had been forgotten. It made him realize that we can't just teach something and then assume that students understand and will remember the concepts.

> I also felt that the learning process for most students is a lot longer than I anticipate and I know this in part having taught classes for many years that things always take longer than they seem on paper. That was very true of this lesson study. I'm not sure if that's because… adult learners learn at a slower pace than some kids who are in class 5 days a week.... How quickly an engaging lesson like this some of this disappears without revisiting the material… When I ask students to remember stuff about the lesson and try activities that were related to the lesson, they couldn't make the connections. They had some difficulty thinking back… So, it made me think that things have to be… revisited. If we really want

> students to get a particular concept, it has to be developed in more than one class. (Ben)

Ben's realization was echoed by Kimia when she talked about the design of the CUNY Framework. There is a need to teach new concepts in a way that connects to what has been taught already, planning time for students to see the same times of content a few times, in different ways.

> I mean, the way the Framework laid it out, the way it rolled out, it seemed so logical to me and I think to them. There was repetition. There were new things. One thing built on each other, but there was enough looping and spiraling that, you know adult ed students don't always come to class, even if they didn't come for a day, or two days in a row, they could still catch up and do the work. (Kimia)

The close observation of students, note-taking on student thinking, and debriefing of the research lessons strengthened a focus on the specifics of learning so that the participating teachers increased their ability to observe students (Lewis, Perry, & Hurd, 2004), recognizing understandings and misunderstandings.

Improving our Curriculum

An important take-away for the whole group was that instruction should be coherent and carefully planned. This means that a series of lessons must be carefully planned across a semester, not just within a class period. However, teachers need resources to make this happen. It's impossible for a part-time teacher to plan instruction in this way, which means that resources have to be provided, such that curriculum appropriate to adult learners is made available to adult

education teachers. Even then, it's clearly not just a matter of putting the lesson plans in the teachers' hands. Even the existence of well-structured curriculum requires planning time and, ideally, collaboration with other teachers to be successful.

Though we had originally intended to fit our lesson study within the functions and algebra curriculum of the CUNY Framework, specifically because of these issues, it again became clear that a lesson study research lesson should fit into a larger unit of instruction, so that students are well-prepared for the content and teachers are not put in the position of trying to cram everything into one lesson. There was general agreement that there is a need for a HSE curriculum framework for geometry.

> I think one of the things that came out of it in our teaching and our discussing of it, is that our students in this particular lesson have to know a few things or be aware of a few mathematical concepts before they can do stuff like this. We introduced things like 3D shapes, 3 dimensions, 2 dimensions, things like that... There needs to be a little bit of prep for the lesson that we did… I thought the process of doing the lesson study helped highlight some of the things I just mentioned. Some of the places where you can go back and develop pre-lessons for it. Maybe make a bridge between some of the Framework and have a few lessons beforehand that connect directly to the lesson study. (Ben)

In the absence of these materials, there was an idea that the teachers would put together a sequence of lessons for the following semester that would lead into the lesson that we wrote for our lesson study.

> So, my thought was how can I adapt what I have learned in this process to the students that I have? How can I modify it? Which is how I thought to extend this lesson over a whole cycle, that's where that came from. How can I adapt this and see if my students can get it? Because it's information that they need, so those were the very real positives that came out of the lesson study. (Diane)

From listening to other teachers speak about the CUNY Framework, Diane also realized that she wanted to try teaching these lessons. She thought it might help with the frustration she has been feeling in her teaching lately.

Challenges in Lesson Study

Time. All of the teachers mentioned the difficulty of making time for lesson study meetings. Teachers who taught in the morning often ran to the meeting, not having had time for much of break. Other teachers ended up working a long day waiting for their evening classes.

> Nine sessions is a lot. We did at least two hours each time and spent eighteen hours, then we had the classes that were being observed, so that's another six hours, so that's 24 hours, and then there was some work done online, so you add another couple hours to that, so all in all, it turns out to be close to 30 hours of commitment time, a full week of work to put together, spread out over a few months. (Ben)

> The thing that got me was the amount of time that it consumed. It was time-consuming. It was deep and it was wide and it took up a lot of time… And it made some very long days on Wednesdays when I was teaching on Wednesday

> night. It kind of beat me up a little bit in that regard, but it was good. I would really have to think hard about whether I would participate in another one. (Diane)

Group planning of the lesson. Though the group got along very well overall, the decision-making required for six people to participate in the planning of every detail of a 3-hour lesson could be difficult.

> A lot of what we were talking about was... speculative, and that's good, but sometimes when you have a lot of people speculating, speculating, speculating, it can feel like you're trying to account for every variable. (Ben)

When the group initially chose the math lesson for our research lesson, we had to make the decision through a vote since we didn't reach consensus. Some of us felt that maybe we hadn't chosen a lesson appropriate for our classes.

> Picking out which one of the lessons to do was one that I just wasn't sure which was the best one. And then when it came to when we did pick one, it seemed so long. The lesson seemed so long, and in my mind, I want it to be the perfect lesson, and there's something here I don't know how to fix it so that it is exactly the way we want to teach it and it's exactly what the students will get out of it… So, when we were going through it and trying to figure out well, do we keep all the cards? Do we not? Just making that decision, how to piece it together. There was this one day, I think it was that which we were trying to do and that was the only day that I got a headache out of doing that. Because we were just going back

> and forth, and all of a sudden, I just got a headache….Well, that was the first time I felt like I wasn't enjoying it. I was like, oh my god, this is too much. It felt like it was too much. (Victoria)

It was a challenging day for a lot of us. The research lesson was coming soon and we had decisions that had to be made, but it was a complicated lesson and the path forward was unclear.

> Diane: This is hard! This is hard!
>
> Mark: But fun? A little bit?
>
> Victoria: I have a headache.
>
> Diane: Hmm. You know, are we there yet? It's hard trying to figure this out and trying to make it make sense and trying to come to some concrete decisions. And then figuring it, what are we supposed to do for our homework to actually put it in context? This is hard.
>
> Mark: Yeah... So, we have 20 minutes? I know you have to go now, right?
>
> Victoria: What are we doing in 20 minutes, because I'm looking at the bus schedule.
>
> Mark: Hmm. I'm, uh, adapting.
>
> Eric: It seems to me that we're using the lesson as written mostly.
>
> Mark: Yeah, but even that. We need to pull this out for our students...
>
> Victoria: Is there any more chocolate over there?

Even though we struggled, there was a feeling in the group that similar to Ben's student, we felt that the process, even when it was challenging, was a "good challenging."

> I remember at some point it was really hard and I was like, how are we ever going to do this, and in a way that makes sense and take it to a place where it's presentable, feeling kind of overwhelmed by it, but then it happened, so we kind of pushed it through, so I really appreciated that. (Kimia)

The Future of Lesson Study at NYCALC

All of the participating teachers reported having a positive experience in the lesson study cycle. Three out of four said that would be interested in participating in another lesson study or leading a lesson study themselves.

> If I had more time, I would even be interested in running something like this. Having been, not just a participant in this lesson study, having known a little about lesson study beforehand, that maybe doing a lesson study will encourage teachers to do this on a more informal basis, to discuss more, to talk more, to share more. I feel that over the last five years or so as I've worked pretty hard to get math teachers to be more open, to meet more frequently and I believe our math teaching staff is a lot better than we were 10 years ago, but I still feel that there are gaps. I still feel that there is not enough discussion back and forth, or maybe not enough understanding of student struggles, that having a lesson study, doing a lesson study, it has other benefits, not just the lesson itself. It can help

> change the teacher's mindset and attitudes about how to teach math. So, I think that doing that with teachers is a long-term benefit. (Ben)

Incorporate this into the Whole Field

A few of the teachers felt strongly that lesson study as a professional development practice should be part of adult education generally. They recognized the particular challenges in making that happen, but spoke movingly of their positive experience with our group.

> I hope this is something that CUNY will incorporate more, that it will not just stop here. I think [it should happen] at different campuses, not just [NYCALC]. I think other teachers would do it, teachers who are committed to their students. I think it's valuable. (Victoria)

The teachers' experience corroborated the idea that teaching practice can be changed from the ground up, by giving teachers the opportunity to look in detail at student thinking, reflect on the experience and share teaching strategies with other teachers.

> I just wish there was a way of incorporating this into the whole field. It's such a rich experience… When I learned that this was part of what Japanese teachers do… it's still incredible to me that this is part of their process and so I'm amazed by that…. I consider myself an experienced teacher, but I feel like just through this process... the way I think about my teaching has shifted, so if nothing else, that's a good justification for doing this, because I think the process itself will lend into people's shifting in practice. And that has ripples, because you work with people in your program. (Kimia)

Discussion

Challenging Our Students, Challenging Ourselves

This case study gives a window into the experience of bringing more abstract, challenging materials to an adult high school equivalency class. Perimeter and area of rectangles have been taught in adult education for years. We have used constructivist approaches to help students gain a conceptual understanding of perimeter as a measure of length and area as a counting of squares as opposed to memorizing formulas and procedures (Hinds, 2006). Understanding comes first and then we build formulas with students using that understanding. In our first meeting, Kimia described how a conversation she had with her class earlier in the day, helping students understand a justification for this approach:

> I knew I was going to teach the Pythagorean Theorem... So, I said, you know, I was thinking about this as I was coming to class, at some point, there were mathematicians who figured out relationships. The Pythagorean Theorem is about a relationship, so Pythagoras figured out this relationship. I said, for you to learn math... I think the best way is for you to experience it. Someone else has already seen that relationship, so you're trying to enter into that, so I think the best way is for you to see it rather than me telling you here's the formula... I'm going to give you stuff to cut up and I'm going to ask you to tell me what you see.

It wasn't our initial intention that the subject matter of our research lesson be based on more advanced parts of the TASC exam, but we decided to focus on geometry first, and the enlargements lesson we decided to use included more content than the teachers were able to

explore in a conceptual way before the research lesson. The geometry lesson we taught in this lesson study was based on content standards that have not traditionally been taught in adult education, involving the volume and surface area of figures such as spheres, cylinders and cones. As we planned the lesson, it was clear that some of the teachers were struggling with the lack of experiential learning and scaffolding involved in the lesson. Though teachers knew this probably wasn't true, the lesson assumed that students already had a basic understanding of the measurement formulas and were now using that understanding to look at the consequences of doubling certain dimensions.

Since the lesson wasn't part of a series on geometry, students would have to struggle through making sense of the formulas without necessarily having a conceptual or experiential understanding what the measurements meant. This was very different than the teaching we have done and recommend. However, there is a tension in the field now between teaching for conceptual understanding and teaching students the wide range of material on the HSE exams. It certainly isn't a choice between one and the other, necessarily, but every teacher is put in the position sometimes of deciding whether to bring some new content to the class that students don't have adequate preparation for and possibly won't be able to understand in a full, conceptual way.

The goal of lesson study is not to produce a perfect lesson, but to learn about teaching and student understanding in our classrooms. Our lesson could have been better, certainly. It may have been more appropriate to choose content standards closer to the current understanding of our students so that the scaffolding required was possible within the class period. However, even if this is true, the group learned a lot from the experience. We learned that students were capable

of struggling productively even when the task felt beyond them at times. We started to wonder whether we might be holding our students back if we always require a careful stepping from each concept to the next.

This tension relates to the research question about how lesson study affects teachers' beliefs about students' abilities. In our case, we learned some specific details about what adult students don't necessarily know. For example, many students didn't understand the use of exponents in the geometrical formulas we were using. Our assumptions that students would understand exponents led them to a situation where they doubled quantities instead of squaring them. Seeing this misconception in action was important for teachers because it showed the consequences of making assumptions about what students know. And there are certainly many things about symbolic notation in mathematics that adult students won't know when they come to our classes. Having the eyes to see where these gaps in knowledge tend to occur will be very helpful for teachers. At the same time, however, we also learned that our students are capable of rising to the challenges we give them. They are capable of more than we sometimes give them credit. They may be able to make connections and work to understand things that will make them push, in effect, the zone of proximal development, especially when we working in teams with other students.

> I think what I do is I spend time scaffolding a lot because I know my students and I know what they know and what they don't know, so I know for example if I'm going to teach something on geometry, for example, I know what I've done with them and where I need to go next… I feel that my students are challenged anyway because what I do with them is not necessarily easy for them, but I think this was

> such a big stretch, so it made me feel I can even challenge them more than I usually do, not to hold their hands as much… I feel less afraid of saying okay well they haven't done this, maybe we'll just try it and see what happens and if I haven't done the scaffolding… I think most of them did that, took the challenge and even though they might have not finished or got anywhere near it, they didn't give up. (Kimia)

At the same time, there was a consensus in the group that this lesson should be broken up so that students could try smaller pieces after more preparation. The teachers who worked with lower-level classes were interested in exploring the idea of splitting the research lesson into a series of lessons on different geometric objects (rectangular prisms, cylinders, spheres, etc.) to be taught over the course of a semester. This would give time for exploration and development of a conceptual understanding of concepts such as area, perimeter, volume and surface area before considering the consequences of enlarging dimensions.

Implications

Our experience with lesson study shows that it helps teachers build knowledge from the particular details of the classroom, moving from the concrete experience of teaching towards abstract principles of effective pedagogy. The value of this work is based in the fact that it starts with teachers and moves outward from the classroom. Instead of talking about teaching in the abstract, lesson study helps teachers work through their own experience.

The experiences of our adult education teachers show the same “pathways to improvement” shown in studies with K-12 teachers (Lewis, Perry, & Hurd, 2004). “Increased

knowledge of subject matter, increased knowledge of instruction, increased ability to observe students, stronger collegial networks, stronger connection of daily practice to long-term goals, stronger motivation and sense of efficacy, and improved quality of available lesson plans" were all evidenced in the results of this lesson study.

If lesson study and peer-led instructional improvement has the potential to be so successful, why aren't they more prevalent in adult education? Currently, professional development hours must be approved by state educational departments. This paradigm assumes that workshops by "experts" are the best way to improve teaching. Lesson study planning meetings are not workshops where experts share knowledge; they are places where teachers build knowledge together. If lesson study and other kinds of peer-led teacher development were supported by adult education administration and approved for professional development credit, it would support a path towards real improvement of teaching in adult education.

Teachers know from experience that every wave of reform will be followed by a new wave a few years later, so they learn to listen politely in required workshops that clearly won't help them with the real challenges they face in the classroom. To improve teaching practice, teachers instead need opportunities to reflect on specific choices at the point of instruction. A workshop can introduce strategies and curriculum, but it doesn't generally change instruction dramatically. That requires being closer to the action.

A top-down approach to improving teaching also runs counter to a long tradition of democratic ideals in adult education and the belief that we all learn from each other as equals in the classroom. Adult education is a field that has been built by people who came to teaching in

order to fight for equal rights and democratic values. They have not generally been professional teachers, but came with the belief that everyone has the right to an education and we all have the ability to become educated. Just as our students come to the classroom with important life experiences, our teachers come with skills and experience and must be invited as equals to improve instruction in their classrooms.

In order to be good teachers, love and commitment are necessary, but not sufficient. We also need to know how to teach skills and help students build knowledge. As new teachers come into our field, we need processes that will help them acclimate, deepen their content knowledge, learn basic teaching skills, and lean on more experienced teachers so that our students make good use of the precious time they have with us and learn what they have to learn. Lesson study provides a way to improve teaching by trusting teachers and giving them the structure to build their own expertise as a member of a respected profession.

As a society, we have increasingly created higher expectations for students and teachers without necessarily giving additional support. It is unreasonable to expect greater outcomes from adult education programs without similar investment in the teachers who help students succeed. The investment necessary to support teachers is not necessarily expensive. A few extra paid hours a week to collaborate with colleagues is not an expensive proposition. What it does require is a shift in orientation about what is valued. The time that teachers spend preparing individually and in small groups must be supported and protected. In exchange, teachers have the opportunity to experience tangible improvements in their teaching practice, with concurrent improvements in their students' knowledge.

If we really cared about the education of adults, we would also commit the resources necessary to give adult education teachers adequately compensated and respected professions. Investment in lesson study as a professional development practice in adult education could be a first step towards that larger goal.

Further Research

The idea of our research lesson as a framework spread out over a semester brought up a couple questions for practitioner research through future lesson study cycles: 1) How could we build a gradual step by step sequence, incorporate manipulatives, and go through all of these shapes with a lower level class? 2) How could we teach these things so students can discover things as opposed to just giving them the formula? How could we incorporate lessons exploring where the formulas come from? Is there a way to evaluate true/false statements about geometric enlargements without the formulas?

There was also interest in continuing the work of our math teacher community. One idea was having a group of math teachers focus on a particular content (perhaps geometry), at least for part of the semester. This would allow for collaboration across different levels and potentially end with materials and a sequence for a framework of lessons and activities. Another idea was to partner teachers over the course of a semester with time to collaborate and offer feedback to each other. A few of the teachers made plans to collaborate in the next semester. Future research might explore whether participation in lesson study increases collaboration among teachers outside the specific context of a lesson study cycle. Does the experience of lesson study make teachers more likely to collaborate in general?

As an example of peer-led instructional improvement based on collaboration and observation, lesson study has been shown to be a particularly effective form of teacher development, but it isn't the only model of its kind. There are possible modifications that may remove some obstacles to implication but still retain most benefits. For example, is it possible to run a lesson study in three or four sessions instead of nine, with only one observation, in order to reduce the cost of paying teachers to meet outside of class? Or might there be a model where a pair of teachers plan a lesson together and observe each other teaching the lesson, with a meeting to make revisions in between? What could a teacher do on their own that might retain some of the useful elements of lesson study? Important elements to retain would be planning for instruction, observation of student thinking, reflection on the experience of teaching and opportunities for collaboration. With these elements in mind, could streamlined models be effective in different kinds of adult education programs? These are all questions that further research might answer.

Summary

By adapting a successful Japanese model for implementing this vision, this case study of a lesson study cycle in adult basic education attempts to capture the experience of using teacher research to improve the teaching of mathematics to adults. In order to change teaching culture, teachers should be supported in using their classrooms as the source of knowledge for improvement. An alternative vision of professional development gives them the tools and the freedom to use this knowledge to increase their students' chances of success.

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Appendix A: Pre-Lesson Study Interview Questions

Interview questions used with teachers before lesson study cycle:

1) How well do you understand the functions and algebra content that is tested on the TASC exam?
2) How comfortable are you teaching the functions and algebra content that is tested on the TASC exam?
3) What is an example of a topic within functions and algebra that you have found difficult to teach?
4) What functions and algebra topics are missing from the CUNY HSE Curriculum Framework?
5) How well do your students understand functions and algebra?
6) What would like to understand more about how students learn functions and algebra?
7) What kinds of professional development have you done in math? What did you like about it? What was missing?
8) What do you know about lesson study? What are the benefits of lesson study? How does lesson study contrast with other professional development you have participated in?

Appendix B: Post-Lesson Study Reflection Questions

Reflection questions for teachers used in interviews after lesson study cycle (Kotelawala, 2012):

1) What thoughts did you have when looking at the transcript of our first conversation before the lesson study? Did anything stand out to you?
2) From the lesson study, what thoughts on student learning and understanding emerged?
3) From the lesson study, what thoughts about your own teaching emerged?
4) What was it like being observed? What was the lesson debrief afterwards like for you? (For two teachers who taught the research lesson)
5) What did you find most challenging in the process of group planning? Be specific, using an example.
6) Which parts of the process did you find most useful and why?
7) Would you be interested in being an observer at a lesson study in the future? What might be benefits of doing so?
8) Do you have any final thoughts on the experience?

Appendix C: Research Lesson Plan

See attached.

Lesson Study Research Lesson - Final

The Goal of Our Research Lesson: *Students will enjoy math as a conquerable challenge by building on successes and developing a systematic approach to problem-solving.*

We want students to see math as a challenge, not as a problem. They will have the confidence to not view struggle as a negative statement about their own ability, but as an expected part of learning. They will use their personal experience and the information given to approach mathematical situations with the spirit of a scientist, with the confidence to move forward and try things that may not work. With each challenge they work through, they will gain confidence in their ability to use a systematic approach as they make sense of unfamiliar situations or problems.

Content Standards:	Mathematical Practices:
• Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (TASC Item Specifications) • Explain volume formulas and use them to solve problems. (College and Career Readiness Standards for Adult Education)	• MP. 1 Make sense of problems and persevere in solving them • MP. 2 Reason abstractly and quantitatively • MP. 3 Construct viable arguments and critique the reasoning of others • MP. 5 Use appropriate tools strategically • MP. 6 Attend to precision • MP. 7 Look for and make use of structure • MP. 8 Look for and express regularity in repeated reasoning

Pre-Assessment: A Fair Deal

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Teacher gives students 15-20 minutes to complete as much of the A Fair Price handout as they can. This will happen in a class prior to the research lesson being observed. After the pre-assessment, look through student work and summarize their difficulties as a list of questions. Students will get their initial assessment with a question/prompt along with a blank assessment to revise their work after the research lesson.	Asking clarifying questions Looking confused Putting their pencils down Being curious Correctly answers all the questions (probably not common) Common Issues: ● Leaves question blank ● Assumes diagrams are accurate representations ● Fails to mention scale ● Focusing on non-mathematical issues ● Makes a technical error ● Simply triples the price of the pizza or doubles the price of a cone of popcorn	If you are not sure how to answer any of the questions, for now, please just try your best to write any ideas you have. During our class on December 29th, we'll be working on a lesson that will help you. After that lesson, you will have a chance to revise your work. Think of this as your first draft.	Though we won't be going over these pre-assessments, students will have the opportunity to revise their work after completing the research lesson. We will return students' work with a question/prompt along with a blank copy of the handout. Does the research lesson impact students' ability to improve their initial response to this activity?[1] Some students will work on both the pre- and post-assessment, some will only work on the post—one thing to consider is… Does working on A Fair Deal before the research lesson impact students' reasoning during the research lesson?
Message to Students before the research lesson: ● A group of teachers have been working together, writing a lesson on some of these ideas. One of our goals is to focus on your learning and become better teachers. This activity will give us a better sense of how we can help you learn some important concepts in math. ● The other teachers in the group will observe the class to see how it goes, since we all planned it together. They are observing to see how well the lesson works, not whether you get the right answer or not. ● We are planning to take notes and record the lesson so that we can meet afterwards to see what worked well and what could be improved. ● Your participation is voluntary. We really appreciate your willingness to help us improve our teaching.			

[1] Research questions are highlighted with a dashed border. Observers should look for and document evidence related to the question.

Launch - 30 minutes

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Put up newsprint with the following vocabulary words on it: perimeter, area, double, volume, circumference **Testing the Validity of Statements** Write the following statements side by side on the board. *If you double the length and width of a rectangle, then you double the perimeter* *If you double the length and width of a rectangle, then you double the area* Ask students: ● *When we talk about the area and perimeter of a shape, what are we talking about?* ● *How do we measure the area and perimeter of a rectangle?* Write the following definitions on the board: ● Perimeter is *the distance around a shape* ● Area is *the size of a surface*	Students may not believe that a square is a rectangle. Students may: ● Define perimeter as "you add up all the sides" ● define area as "length times width" ● define area as "the space inside"	Square vs. rectangle: ● These are the characteristics of a rectangle: ○ Opposite sides are equal. ○ 90 degree angles in the corners ● Does a square fit these requirements? ○ Yes. A square is a special kind of rectangle.	The rectangle work at the beginning does a few things: ● Lets students know they can use manipulatives and drawings ● Lets students know they can just try different numbers ● Models evaluation of statements, and revising a false statement to be true ● Provides the form of the If/Then sentence. This introduction provides students with a model of how they should work during the collaborative task. Will students use manipulatives to see the change in perimeter and area when doubling length and width of a rectangle?

Ask students: *What does it mean to double something?* Under each statement on the board, write T F ? Ask the class to vote on each statement - do they think it is true, false, or "I'm not sure". This should take less than 1 minute. Don't give them too much time to think/calculate. Give out handout to students with the statements. Hand out manipulatives (tiles), grid paper and rulers. Encourage students to use these materials when they evaluate the statements. Say: *I would like you to decide whether each of these two statements is true or false. For each statement, write a convincing explanation. If you think a statement is false then replace it with a correct statement.* Students should work independently for 3 minutes before sharing with a partner. Look for the following solution methods for evaluating the rectangle T/F statements: Used tiles or other manipulatives to build two rectangles with appropriate	Students may raise the issue of squaring/numbers raised to the second power Many students might think both statements are true. Some will be unsure. Some students may: ● struggle to understand what they're being asked to do. ● not want to "write on the board", and prefer that the teacher do the writing. ● not volunteer their answers ● not even try to accept the challenge ● guess (correctly or incorrectly) instead of reasoning/experimenting ● express that they "can't draw" ● not be able to explain their answer ● not be able to rewrite false statement ● confuse area and perimeter ● may double length but not width, or vice-versa. Questions students may ask: ● Can I/we use a calculator? ● What size is the rectangle? ● Will this be on the test?	For students who struggle to get started: ● Suggest they draw a picture of a rectangle ● Ask the student to assume side lengths for the rectangle. What could the length and width be? See what happens. For students who struggle to explain their reasoning: ● Can you draw a picture to convince me? ● Can you give some examples to convince me?	Will students make drawings to see the change in perimeter and area when doubling length and width of a rectangle? Will students try different numbers (like scientists!) to see the change in perimeter and area when doubling length and width of a rectangle?

length and width. Drew two separate rectangles with appropriate length and width. Compared the perimeter and area of two rectangles using numeric values for length and width. Drew two rectangles, the original inside the larger rectangle, so that the original is seen as ¼ of the enlarged rectangle. After 5-7 minutes, ask for a thumbs up from anyone who used the tiles or graph paper. Ask a volunteer to explain what they did to evaluate the perimeter statement. Then ask for a thumbs up from anyone who used numbers to evaluate the statement. Ask a volunteer to explain what they did to evaluate the area statement. Don't present algebraic explanations yet. After the presentations, ask the rest of the class: *How do we know for sure that these answers are correct for all rectangles?* Conclude and put up newsprint with: *If you double the length and width of a rectangle, then its perimeter is multiplied by 2.* *If you double the length and width of a rectangle, then its area is multiplied by 4.*	• How much time do we have? • Can I have more time? • Is this right/correct? • Do we have to do the math for each one?		

Problem Posing - 10 minutes

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Organize students in groups of two or three. Hand out calculators. Give out formula sheet. Ask students to look over the sheet, focusing on the rectangular prism and the circle. Ask them to write down two things they notice and what questions they have. Put up newsprint with rectangular prism and circle Ask them to share what they notice and their questions. Definitions: ● **Volume** is the space inside an object ● **Circumference** is the distance around a circle (related to perimeter). Give out Problem-Solving Instructions. Read instructions aloud. What am I asking you to do? Who can put this in their own words? Give each group a set of cut-up True or False cards for the statements for the rectangular prism and circle, a piece of chart paper, a marker and a glue stick.	Students may not be able to identify each shape with correct mathematical term. Possible confusions: ● *lwh* ● *What is r?* ● $2\pi r$ *versus* πr^2 Students might be confused between the difference between a rectangle and rectangular prism, circle and sphere.	Share the correct mathematical term for each object. What does *lwh* mean? ● It means to multiply the length (*l*) by the width (*w*) by the height (*h*) What is the difference between $2\pi r$ *versus* πr^2? Which one involves doubling? Which part of the circle do they each describe? What is the difference between the surface area and the volume of a rectangular prism? What is the difference between a rectangle and rectangular prism? What's the difference between a circle and a sphere?	Students should understand that three-dimensional shapes can be measured in terms of length, area and volume. Students should understand the elements in the geometric formulas for rectangular prisms and circles to be used during the problem-solving time. Will this step improve student use of the appropriate formula while evaluating the statements?

Student Problem Solving - 60 minutes (45 min to evaluate statements, 15 min to compare their work)

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
During small group work and problem-solving: ● Take note of student approaches to the task ● Support students without doing the thinking for them. **Look for the following solution methods:** ● Builds rectangular prisms with *manipulatives* to see what happens when different dimensions are doubled. ● Draws *diagrams* of objects to see what happens when different dimensions are doubled. ● Uses *formula with sample dimensions* for original object before scaling. ● Note groups that use any of the above methods, but go further and compare original and scaled measurement to determine scale factor. ● *Use of* π ○ *Retains* π *in calculations*, e.g. compares 16π and 64π. Compares original and scaled number multiplied by π to find	Faster students might work past their group members Some students might struggle to get started.	Encourage students to engage with each other's explanations and take responsibility for each other's understanding. ● *Judith, why do you think this statement is true/false?* ● *James, do you agree with Judith? Can you put her explanation into your own words?* If students are struggling to get started on the task: ● Ask which strategies did you do when evaluating the two statements about the rectangle? ● Have them draw a picture of the figure and label the parts of the shape. Ask "How can you test this statement?" We want to invoke the answer "By using numbers." If students are struggling midway: ● *What formula can you use to check if the statement?* ● *If it is not twice as big, by what factor has the area/volume increased? How do you know?*	Will students remember what these measurements mean when they use formulas? Will students draw rectangular prisms in order to visualize the effect of doubling dimensions? Will students retain pi for their measurements or will they substitute an equivalent like 3.14? Will students disregard numbers to the right of the decimal point when comparing sizes of measurements, e.g. area of two circles: $(3.14159 * (4)^2 = 50.26544$ and $(3.14159 * (8)^2 = 201.06176$? Do we need an activity like the following to help students prepare for dealing with decimals? Will students understand why it's okay to retain π in their calculations?

scale factor. ○ Calculates measurements using 3.14 or other approximate, e.g. $\pi(4)^2 = 50.24$ and $\pi(8)^2 = 200.96$. ● *Algebraically compares* two versions of formula to see the effect of scaling, e.g. πr^2 and $\pi(2r)^2 \rightarrow \pi 4r^2$. Finds the scale factor in the second equation.	Issues with doubling: ● Students might not understand doubling. ● They may double only one measurement when there are multiple measurements to double (i.e.- the student may see the word *double* and think "times 2" for each one). Students might use an inappropriate formula Students may "get lost" in their doubling and evaluate a statement incorrectly by comparing the wrong numbers. Some students might move through the task more quickly than others.	Focus student attention on all the words on the slip. Refer students back to definition of double from the board. Ask: ● What does it means to double something? ● What measurement did you double? ● What should be the total number of doublings you need to do for this one? How does it differ from others you have doubled? Ask group to read statement out loud. Ask what part of the shape the statement is referring to. Ask which formula describes that measurement. Can group describe/label the numbers in their work. Which numbers are they comparing when evaluating the statements? Hand out cards for the other 3D shapes. Ask: *What happens if the phrase 'multiply by 3' replaces the word 'double.'* Get them to work together by posing questions such as above to get started or assign them each tasks such as one person draw, the other person choosing the formula needed and deciding how to incorporate the formula.	

After 45 minutes, call the class together. Ask them to flip the Problem-Solving Instruction Sheet. On the back are the questions for comparing their poster with the work of a neighboring group.	Some groups may want to continue working on their posters.	If one group is particularly far behind, teacher may allow them to continue evaluating statements while the rest of the groups compare their work.	

Whole Class Discussion - 20 minutes

Steps/Questions in activity	**What are students doing? Expected responses/reactions**	**Teachers' response to student reactions**	**Goals/Method of Evaluation/Assessment (For each step)**
Refer students back to the board notes from the problem-posing section: *If I double the length and width of a rectangle, then the area is multiplied by 4.* Tell students: We are going to look at a chart that follows the same structure of this sentence. It includes all the statements on the cards that you've been working on. Share the handout, ***What happens when we double dimensions of different shapes?*** Ask students: What do you notice? Take a few minutes on your own to write things you notice. Pair/share for a couple minutes. After five minutes, record student responses on the board.	Students will notice a range of things in the chart. Some students may be overwhelmed by the amount of information in the chart. During sharing of noticing, some students might share an inference.	For students who are overwhelmed: ● *How does this chart work? Can you read the first few lines for me?* ● *Does any information here support the your groups evaluations?* ● *What patterns do you see?* ● *Look at all the times when something is multiplied by 2 or by 4 or by 8? What do those situations have in common?* ● *(pick a row) Which dimensions changed in the first column? Circle those dimensions in the rewritten formula column. Do that for a couple and see if you notice any patterns,* Ask students who share an inference to explain what they say in the chart that helped them to notice that.	How well do students listen to each other? Are students able to demonstrate understanding of other students' reasoning? Do students see any connections between the formulas (in expanded form) and the scale factors? What patterns do students notice?

Summing Up - 10 minutes

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Looking back at the work you did today, we can see that you have calculated the perimeters, areas and volumes of different shapes after scaling them to larger sizes. You have demonstrated that you know how to use the formulas for a rectangular prism, circle, sphere, cylinder, and cone. These are important skills for the TASC exam. You've developed strategies for figuring out if a mathematical statement is true/false. One important take-away is that doubling one or more dimensions of an object doesn't necessarily double the perimeter, area, or volume (measurements) of that object. *What's an example where doubling one or more dimensions doesn't double the perimeter, area, or volume?*	Sharing and listening Students cite the 2nd or 3rd rectangular prism statement or the 2nd circle statement. Students cite examples based on shapes and statements they did not evaluate during the problem-solving	If necessary, point to student examples.) For example, we saw that doubling the radius of a circle makes the area 4 times the original area. Doubling the radius of a sphere makes the volume 8 times the original volume.	

Reflection - 5 minutes

Hand out the final reflection.		If there is no time for reflection, ask "How did it feel working on today's lesson?"	

Post-Assessment

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Return to students their original responses to the *A Fair Price* assessment task as well as a second blank copy of the task. If you have not added questions to individual pieces of work then write your list of questions on the board. Students are to select from this list only the questions they think are appropriate to their own work. *Look at your original responses and the questions (on the board/written on your script).* *Answer these questions and revise your response using what you have learned to improve your work.*	Students may want to take the assignment home or complete it in the next class.		Will there be time to complete the post-assessment during the class period?

Boardwork

List of vocabulary to be defined during lesson. Statements about rectangles to be evaluated Student vote	Student Work *If I double the length and width of a rectangle, then the perimeter is also doubled.* (Original statement is true) *If I double the length and width of a rectangle, then the area is multiplied by 4.* (Re-write of false statement)	Notice/Questions about formula sheet	What do we notice? - Chart

Materials

- Newsprint with concluding statements from the end of the launch
- Blank newsprint (one for each group)
- Markers
- Color tiles
- Graph paper
- Rulers
- Calculators
- Newsprint with circle and rectangular prism, diagrams and formula
- Glue sticks
- **Handouts:**
    - A Fair Price (a clean copy and a copy of each student's initial assessment)
    - Launch Two Statements on Enlarging Rectangles
    - Group instructions for problem-solving task / Instructions for Looking at Other Group Work
    - The formula sheet (one for each student)
    - True/False cards (cut up, divided into shapes, enough for one set per group)
    - Whole Group Discussion: Chart - What happens when we double dimensions of different shapes? (one for each student)
    - Learning Reflection