Area and Perimeter: Reasoning Beyond Procedures

by Patricia Helmuth

Researchers who have conducted studies on students' understanding of area and perimeter have found that it is very common for students to mix up these two measurements of a rectangle as well as fail to grasp their inherent relationships (Sanfeliz, 2023). Consider as an example, one study that was conducted to assess Grade 10 learners' conceptual understanding of area and perimeter. Overall, students answered 50% of the assessment questions correctly, but when students were asked to define area and perimeter[1], 67% of the students could not define area and 90% could not define perimeter (Machaba, 2016).

In analyzing why students confuse the concepts of area and perimeter, an overreliance on and the premature introduction of formulas has been frequently cited as contributory factors (Walton & Randolph, 2017). If students "learn" procedures for calculating area and perimeter before they gain conceptual understanding of either, students might easily confuse the two formulas since they both require students to do *something* with length and width[2]. To add to that conundrum, in *A Pathway to Understanding Area and Perimeter*, Sanfeliz (2023) concluded, "when students memorize formulas without the understanding of the concepts, they have difficulties generalizing procedures." This suggests that if students do not have a solid grasp of the underlying relationships between area and perimeter, this may serve as a barrier for students in understanding what surface area is, or how volume relates to area and perimeter, and may block entry points to many other geometry and algebra topics that require students to apply knowledge of those concepts in a different context.

This lack of conceptual understanding can lead to more than simply making errors in calculations when solving area and perimeter problems. In fact, when a grasp of fundamental concepts is lacking, students commonly develop misconceptions about area and perimeter, which is a bit more troublesome. A misconception is based on faulty conceptual understanding and not "easily corrected by telling the learners that they are wrong" (Machaba, 2016). For example, students may reason that as the perimeter of a rectangle increases the area increases as well (Liping, 2010). Since this conjecture is sometimes true, as illustrated in Figure 1, students would benefit by engaging with authentic learning experiences that challenge this assumption, or as Machaba puts it, "learners need experiences that will enable them to reorganize their thinking."

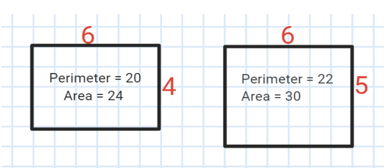

Figure 1

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[1]Area is measured in square units (square inches, square feet, square meters). When we measure area, we are essentially counting squares. Perimeter, on the other hand, is a measure of length and is measured in linear units (inches, feet, meters). Instead of counting squares, students should imagine measuring around their rectangles with a ruler or tape measure (Trushkowsky, M. 2015).

[2]The relationship of length and width to area is multiplicative, while the relationship of length and width to perimeter is additive (Liping, 2010).

One way to help students to reorganize their thinking is to provide them with learning experiences that enable them to explore the relationships of area and perimeter within one high-cognitive demand math task[3]. It is not enough to show students a rectangle, talk about the relationships between perimeter and area, and model the way to solve area and perimeter problems using two different algorithms. Even supplying students with a context, such as the perimeter and area of a basketball court, may not be tangible enough for students to comprehend and take hold of the underlying concepts (Kaplinsky, 2017). A more authentic experience is required. One way to help students to reorganize their thinking, would be to supply them with opportunities and the tools to physically manipulate and reorganize the areas and perimeters of rectangles.

I had the privilege to be a guest teacher in Eric Appleton's online CUNY Math Problem Solving class where he created an activity for his students to complete in Mathigon Polypad . He tasked students with arranging 36 tiles into one or more rectangles (Figure 2). Try it yourself by following this link. As you work through it, think about how you might use this activity to help your students to gain conceptual understanding of both area and perimeter.

What did you discover?

What might your students discover as they work through this activity?

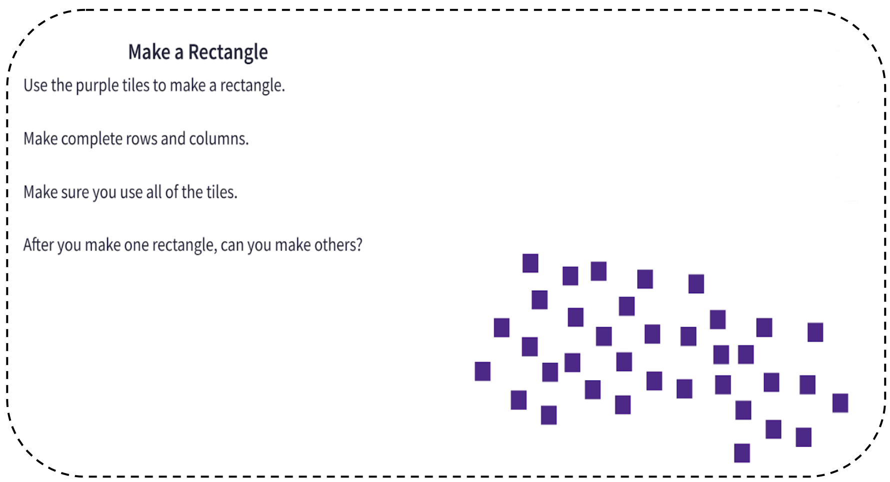

Figure 2

Take a moment to examine this example of student work before continuing with this article.

What do you notice?

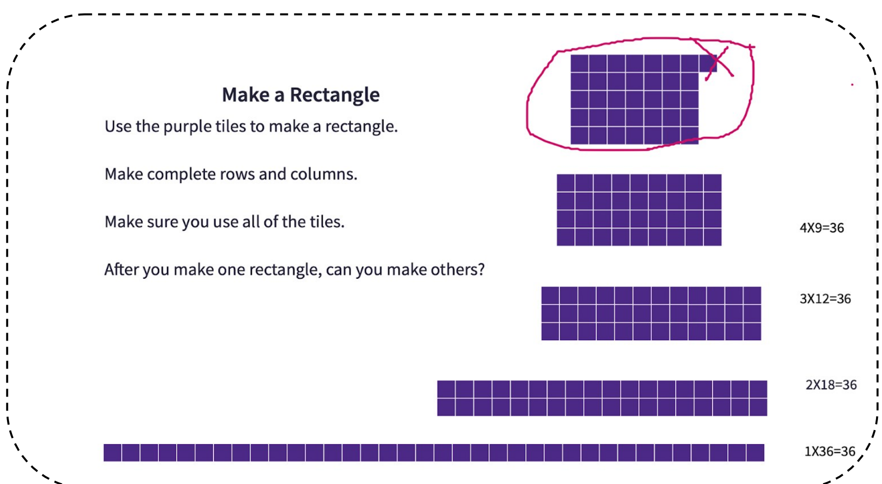

Figure 3

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[3]High-cognitive demand math tasks require students to think deeply about a mathematical situation. The problems are open-ended, meaning there is more than one way to solve the problem and there may even be more than one solution. The purpose of these tasks is to provide space for students to reason through a math situation and develop their own problem solving strategies.

Notice that by reorganizing the squares this student discovered some big ideas:

- There are at least four rectangles that have an area of 36 square tiles.
- Neither 5 nor 7 will work as a side length to create a rectangle that has 36 square tiles.
- There are at least eight factors of 36.
- Multiplying adjacent sides of the rectangles will result in the total number of square tiles.
- All of the rectangles have the same area but different perimeters.
- Adding up all four side lengths will result in the total perimeter.

To draw out these big ideas in classroom discussion, get students talking about their rectangles by asking questions, such as:

- Who would like to describe the rectangles you found?
- Is it possible that we missed one? (You may have noticed that 6 x 6 is missing from Figure 3. In my classroom, I've noticed that students often miss 1 x 36).
- What do these rectangles all have in common?
- How do you know they all have 36 squares?
- How could we find the area of the rectangles without counting each small square inside the rectangle?

The goal here is for students to verbalize the strategies they used to find the rectangles, describe any relationships they noticed, and share any informal methods they discovered to figure out area and perimeter. For example, it is likely that at least one student will be able to talk about how the area can be found by multiplying two side lengths. The student may say something like, "If I multiply one side of the rectangle times the other side, I get the area." With this as a foundation, you can ask additional questions to get students thinking about how to clarify and generalize the methods they have discovered, such as:

- Can you find the area if you multiply two opposite sides?

To find the area of a rectangle that is not a square, students may use language like, "It has to be the top side of the rectangle times the side of the rectangle." Given this response, you might write on the board: **top side x side = area**. Ask if anyone else has another way to describe the sides of a rectangle. Perhaps a student will remember **length x width**, which is ideal, because that becomes shared knowledge generated by student discussion rather than teacher talk. If that happens, be sure to validate any other words students use to describe the sides of rectangles as well as any informal methods that students share to find the area of a rectangle or square (a special kind of rectangle). The purpose of validating students' own language is to use it to form a bridge from student observations and language to the math vocabulary and procedures that will be useful for them, not only when they take tests, but in their everyday lives.

When students have agreed on a method that works to find the area of any size rectangle or square, they are ready to work with rectangles that do not have a visible grid. They will understand that the product of **length x width** represents the number of squares that cover the interior surface of a rectangle, even when those squares are unseen (Trushkowsky, 2015).

A similar line of questioning can be used to talk about perimeter, starting with what students intuitively noticed about the perimeters of the rectangles they created and the informal strategies they used to find the changing perimeters. Questions such as:

- What do you notice about the perimeter of these rectangles?
- How can we find the perimeter of each of the rectangles without counting every small square on all four sides of the rectangles?
- What relationships do you see between the numbers we use to find the area and the numbers we use to find the perimeter of rectangles?

This line of questioning is meant to draw out what students have observed in the concrete visual models of small squares that make up variously shaped rectangles, to start with the exact words students use in their descriptions and informal methods, and then to bridge from students' conceptual understanding to more abstract thinking.

Look for examples of student work that can deepen conceptual understanding of perimeter. For example, if we circle back to the student who created the 5 x 7 + 1 shape (Figure 3), we can shine a light on this as a wonderful example to think about perimeter.

- What is the perimeter of that shape? What is the area?
- What would happen to the perimeter if we took away that extra square on the top right?
- What would happen to the perimeter of *any* rectangle if we took away the four corner squares (Figure 4)?
- What would happen to the perimeter of a rectangle if we take away four squares from the length of the same rectangle (Figure 5)?

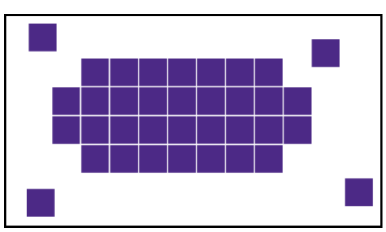
Figure 4

Subtract 4 corner squares

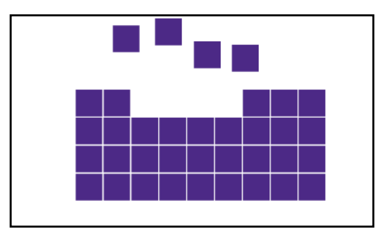
Figure 5

Subtract 4 squares from length

Have students make predictions, explore, and form generalizations about how to find the perimeters of the composite shapes[4] that they create.

[4]A composite shape is the combination of two or more basic shapes. Examples would be two different rectangles that are combined to make an L shape or three rectangles that are combined to make an H shape.

When doing this activity in a classroom setting, students might be given square plastic tiles to physically move around to create rectangles and then asked to compare their rectangles with what others. Graph paper also works really well in an in-person classroom if square tiles are not available. Have students create as many rectangles as they can (for a fixed area) on the graph paper. In addition to giving students 36 as the fixed area, try using 42 when using graph paper because it gives students an opportunity to practice the same skills while also drawing out other multiplication facts, such as 6 x 7 = 42.

Finally, there are many other activities that can be used to deepen students' conceptual understanding of area and perimeter and their relationships. In the activity previously discussed in this article, a fixed area was introduced to encourage students to think about changing perimeters. Alternately, students could begin with a fixed perimeter to promote thinking about changing areas. In this issue of *The Math Practitioner,* you will find one such activity that is featured in the CALM curriculum*: Understanding Perimeter with Formulas.* For a complete lesson plan with the answer key, click here.

Please consider sharing your classroom stories about area and perimeter! What activities do you use with your students to help them gain conceptual understanding of area and perimeter? Contact the editor of this newsletter for information on how to share your student stories with the ANN community: mathpractitioner@gmail.com.

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Garden Fence Challenge
U1.L4

I have 100 feet of fencing.

I want to make a rectangular garden that has a fence all the way around it.

What size will the garden be?

Show how you know you will use all 100 feet of fencing.

Extra: How much space will I have in my garden?

Source: [TERC CALM Curriculum]