

*Problem Posing  
and Problem  
Solving in a  
Math Teacher's  
Circle*

By

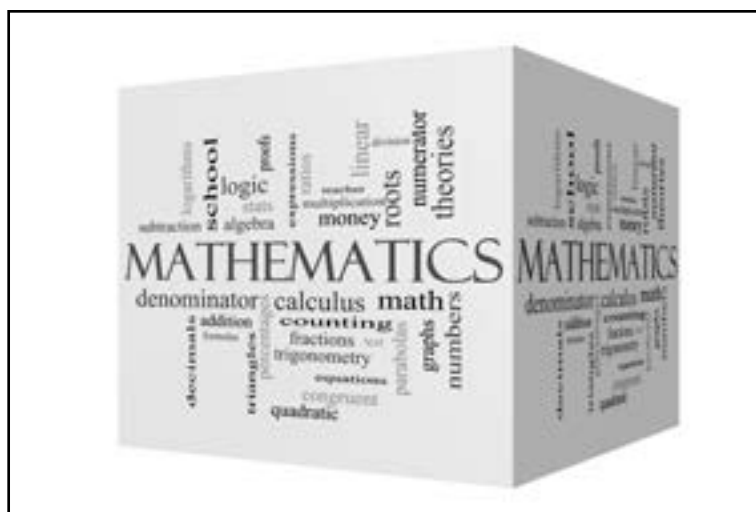
*Eric Appleton  
City University of New York*

*Solange Farina  
Borough of Manhattan  
Community College/ City  
University of New York*

*Tyler Holzer  
Fifth Avenue Committee*

*Usha Kotelawala  
City University of New York*

*Mark Trushkowsky  
City University of New York*



**Abstract**

This article describes the New York City Community of Adult Math Instructors (CAMI), a math teachers' circle founded in November 2014. The authors share details about their own participation in CAMI to show the professional growth that research-based, peer-led professional development can offer for adult educators.

Adult educators are often expected to teach a wide range of subjects, but generally do not have formal training in mathematics or mathematics education. According to Ginsburg (2011), “few teachers begin their adult numeracy teaching with the skills and knowledge needed to design engaging, effective instruction.” Ginsburg goes on to make a case for content-based professional development that is rooted in active learning and ongoing collaboration. In this article, we describe the activities of a math teachers' circle organized by adult education teachers.

There is a tradition of dedicated teachers coming together to

form learning communities. ASolange Farina co-founded the Math Exchange Group (MEG), a teacher collaborative of adult educators in New York City that met to do math and improve math instruction from 1993 until 2012. (Brover, Deagan, & Farina, 2000). Math teachers' circles often provide a space for teachers to work on non-routine problems for which solution paths are not always clear (Fernandes, Koehler and Reiter, 2011; Geddings, White, & Yow, 2015). As a professional development opportunity, these circles encourage content exploration and connected pedagogical conversations (White, Donaldson, Hodge & Ruff, 2013). They have been shown to be effective in providing support for teachers, promoting the use of problem-solving as an approach to teaching mathematics and even changing teachers' views of what mathematics is (Donaldson, Nakamaye, Umland, & White, 2014).

### **Context**

This article discusses the professional development approach used by the New York City Community of Adult Math Instructors (CAMI), of which the co-authors are members. Founded in November 2014, CAMI is a peer-led group of teachers from adult basic education, high school equivalency and college transition. Our teachers come with varied mathematical content knowledge and teaching experience. Some have taught mathematics for years. Other members are relearning mathematics they haven't seen since high school. Very few CAMI members have degrees in mathematics or math education. An average of about eight teachers are present at each meeting and, over the last two years, more than 50 teachers have come to at least one. In general, leadership of the meetings rotates among an informal group of eight members, including the authors.

### **Problem Posing and Problem Solving**

When preparing for meetings and choosing activities to explore, CAMI facilitators are guided by the definition of a mathematical problem as a "task for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific 'correct' solution method" (Hiebert et al., 1997, as cited in Van de Walle, 2003). We seek out problems that have multiple entry points and are accessible to a wide range of learners, while also allowing for extensions into more advanced mathematics.

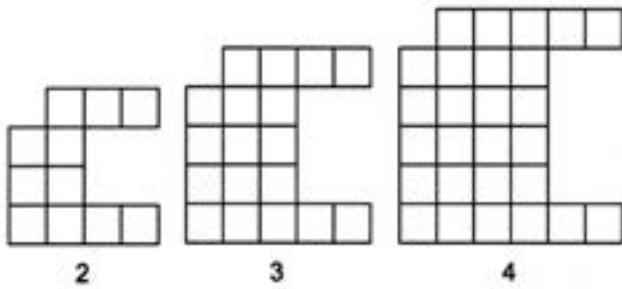
During our meetings, we practice two aspects of teaching and learning mathematics: problem posing and problem solving. We generally start meetings by asking participants to consider a problem and pose questions that come to mind. We then brainstorm more questions in pairs, discuss them as a group and choose questions that we try to answer. We work individually for a while in order develop our own ideas, then work in small groups before presenting solutions to the whole group. The structure of our meetings consists of problem posing, problem solving and presentation of solutions. We base this approach on the teaching of mathematics for understanding through problem solving (e.g., Hiebert et al., 1997), as well as the success of math circles mentioned above.

To illustrate how we use problem posing and problem solving to provide learning opportunities for teachers, we describe a meeting facilitated by Usha Kotelawala. The other authors, along with more CAMI members, participated in the meeting and are quoted below.

### **A CAMI Meeting**

In February 2015 Usha led us through a problem posing activity (Brown & Walter, 2005). She asked us to explore the series of images below (Billings, 2008).

Instead of giving us a specific question to answer, she said: “What do you see? Pose a few questions.”



Individually, we worked for a few minutes to generate questions. This task of coming up with questions, but refraining from working on solutions, proved to be challenging for some. Solange started to make a table of numbers and look for a rule to find the number of squares in any figure. As Usha was walking around to see the questions that teachers were writing, she stopped to talk to Solange.

**Usha:** “What are you working on?”

**Solange:** “I want to know the number of squares in the  $n^{\text{th}}$  figure.”

**Usha:** “Interesting question. Is that the only question we could pose? For now, let’s just focus on asking questions. We’ll look for answers later.”

Participants continued to generate questions. We then discussed them in pairs and posted our favorites on chart paper at the front of the room. Our questions included the following:

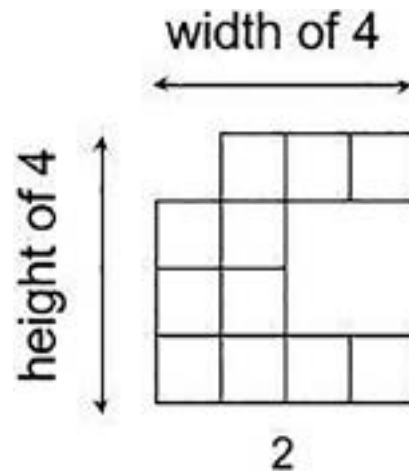
- How many squares are in each figure?*
- What does figure 5 look like? Figure 10? Figure 100?*
- Would figure 5 have an even or odd number of squares? What about figure 10?*

- What is the perimeter and area of each figure?*
- How do the perimeter and area grow for each new figure?*
- What can we learn by exploring the negative space as the figures grow?*
- What is the function for the relationship between the figure number and the number of squares?*

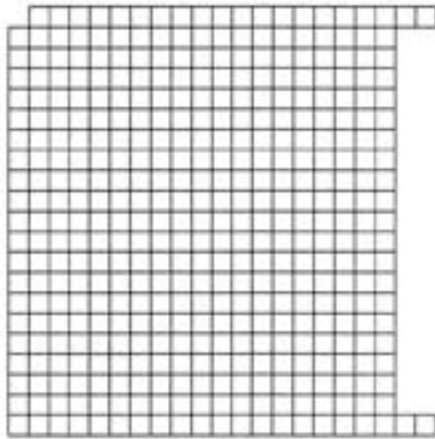
Next, Usha had us reflect on the question-generating activity. This activity allowed us to appreciate how many different kinds of questions can be posed, and, because they came from us, we were invested in answering them. After our discussion Usha had us work together with a partner on a question that interested us. What follows is a description of three presentations shared towards the end of the meeting.

**Avril and Mark** chose to work on the question: *How would you describe the 19th figure so that someone else could draw it?*

Avril created a chart focusing on the height and width of the figures. She noticed that the height and width of each figure is always two more than the figure number. So, for figure 2, the height and width are both 4.

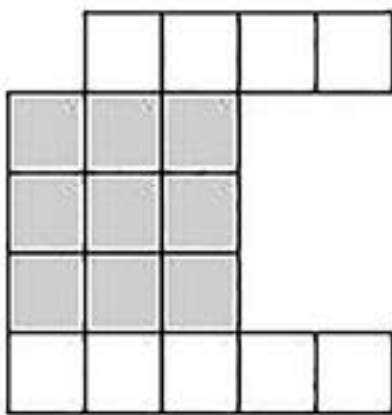


From that, she was able to construct the 19<sup>th</sup> figure. Extending her method, she knew that the height and width had to be 21.



*The 19th Figure*

Mark wanted to answer the question without using an equation. He started off by looking at the three given figures and seeing what kinds of patterns he could find. He broke each figure up into three parts: the top row, the bottom row, and the square in the middle.



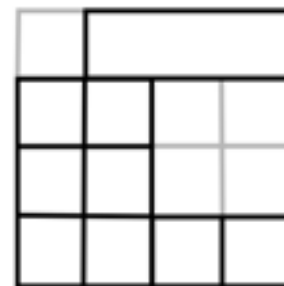
3

*Mark's Square in the Middle*

He noticed that the top row is always one more than the figure number and that the bottom row is two more than the figure number. He also noticed that the middle was always a square with sides equal to the figure number. Mark used these patterns to write step-by-step directions clear enough for anyone to draw the 19<sup>th</sup> figure. Avril and Mark both saw the figures differently, but their approaches complemented each other and they were both able to describe the 19<sup>th</sup> figure.

**Solange and Eric** were interested in the question: *What can we learn by exploring the negative space as the figures grow?*

Similar to Avril, they first imagined a larger square defined by the height and width:  $(n + 2)^2$ , where  $n$  is the figure number. Then they looked at the squares that were missing from the larger square. They discovered a constant of one missing square in the top left corner of the larger square and a missing rectangle on the right side, which could be described as two times the figure number, or  $2n$ . From this way of seeing, Solange and Eric developed a rule for finding the number of squares in the  $n^{\text{th}}$  figure: the larger square  $(n + 2)^2$ , minus the missing rectangle  $(2n)$ , minus the constant missing square  $(1)$ —or, as an algebraic expression,  $(n + 2)^2 - 2n - 1$ .



*Negative Space in Figure 2*

**Tyler, Ida, and Alison** worked on the questions: *Would figure 5 have an even or odd number of squares? Is there a way to figure out if the number of squares in a given figure will be even or odd?*

These teachers saw that the total squares for the three given figures was alternating odd, even, odd, so they made the generalization that the pattern would continue and the fifth figure would have an even number of squares. They drew the fifth figure and counted its squares to make sure this was true.

Figure	# of Squares
2	11
3	18
4	27
5	38

Tyler made a generalization that allowed him to calculate the number of squares in any figure:  $n^2 + 2n + 3$ . He then used the expression to explain why an even-numbered figure will always produce an odd number of squares and vice versa. “If the figure number  $n$  is odd,  $n^2$  will be odd because the square of an odd number is always odd. Two times  $n$  will always be even. Three is always odd. An odd plus an even plus an odd will always be even.”

In her facilitation, Usha asked the three groups to present in a particular order, moving from the concrete to the abstract (Smith & Stein, 2011). We discussed how teachers can use this strategy to orchestrate productive discussions of different problem-solving approaches in their classrooms. We also considered questions that arose from our experience as learners: How can we give our students more time and space to engage with each other’s thinking? How can we help our students adjust to the discomfort of non-routine problems?

## Supporting Teachers

At a recent CAMI meeting, members wrote about CAMI and how it has impacted them both as learners and as educators. One member explained that participating in CAMI has enriched her own mathematical learning: “I’ve deepened my mathematical understanding by working on problems that push the boundaries of the math I know, and I’ve learned so much from seeing other teachers’ approaches to problem-solving.” Another member pointed out that CAMI puts him in the position of being a student: “I have that moment where I get anxious and say, ‘She gets it but I don’t get it,’ and it’s that feeling that our students face every day.”

CAMI helps teachers feel supported in an increasingly test-driven adult education landscape in which conceptual understanding is often passed over in favor of teaching procedural skills. One member explained that much of her time now involves monitoring Test of Adult Basic Education (TABE) results, and so her time spent at CAMI meetings is refreshing. As another teacher wrote, “It’s really encouraging to remember that there are so many teachers out there engaged in the same struggle. . . I need constant reminders not to try to cover everything.”

## Final Thoughts

The ability to teach math improves as content knowledge grows (Harel, 2008). In order to improve mathematics teaching in adult education, teachers must have positive experiences learning the math they are teaching now, and then reflect on that learning.

CAMI provides a space for teachers to become learners and model the learning environment that we want to create for our students—an environment that few of us had in our own math education. For

many of us, CAMI is the math class we wish we'd had when we were in school and the one we would like to give our students: a place where all voices are heard, where different levels of mathematical experience are welcome, where persistence, curiosity, and elegance are valued in equal measure, and where you formulate your own thinking and learn from the thinking of others. CAMI is a sustainable and replicable model of professional development that impacts us in our roles as teachers *and* learners.

For help starting a math teachers' circle, visit our website—[nyccami.org](http://nyccami.org)—for math problems, solution methods and discussion notes from our meetings. ❖

---

**Eric Appleton** is a teacher/professional development coordinator in science and mathematics for the CUNY Adult Literacy and High School Equivalency Program.

**Solange Farina** is a teacher/instructional coordinator in the Adult Basic Education Program at the Center for Continuing Education and Workforce Development for the Borough of Manhattan Community College/CUNY.

**Tyler Holzer** is the high-school equivalency coordinator and teacher at the Fifth Avenue Committee, a nonprofit agency serving Brooklyn, NY.

**Usha Kotelawala** earned her doctorate in mathematics education after eleven years of teaching secondary mathematics and coaching teachers. As the Director of Mathematics Programs at CUNY's Collaborative Programs, she leads curriculum projects and professional development for New York City teachers.

**Mark Trushkowsky** is a teacher/professional development coordinator in mathematics for the CUNY Adult Literacy and High School Equivalency Program.

## References

---

- Billings, E. (2008). Exploring generalizations through growth patterns. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics: Seventieth yearbook* (pp. 279–293). Reston, VA: National Council of Teachers of Mathematics.
- Brover, C., Deagan, D., & Farina, S. (2000). The New York City math exchange group: helping teachers change the way they teach mathematics. *Focus on Basics*, 4(1), 1-15.
- Brown, S. I., & Walter, M. I. (2005). *The art of problem posing* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.
- Donaldson, B., Nakamaye, M., Umland, K., & White, D. (2014). Math teachers' circles: partnerships between mathematicians and teachers. *Notices of the AMS*, 61(11).
- Fernandes, A., Koehler, J., & Reiter, H. (2011). Mathematics teachers circle around problem solving. *Mathematics Teaching in the Middle School*, 17(2), 108-115.
- Geddings, D., White, D., & Yow, J. (2015). SCHEMaTC: The beginnings of a math teachers' circle. *The MathMate*, 8.
- Ginsburg, L. (2011). Adult numeracy demand & provision. A Reader, Four Papers from CAAL's January 2011 Roundtable on Adult Numeracy, 1-27. Council for Advancement of Adult Literacy. Retrieved from <http://www.caalusa.org/Adult.pdf>
- Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. Proof and other dilemmas: Mathematics and philosophy, 265-290.
- Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.
- Trushkowsky, M. (2015, February 10). Problem-posing with visual patterns. [Blog post]. Retrieved from <http://nyccami.org/problem-posing-with-visual-patterns/>
- Van de Walle, J. A. (2003). Designing and selecting problem-based tasks. In Lester, F. (Ed.) *Teaching mathematics through problem solving prekindergarten - grade 6* (pp. 66-80). Reston, VA: National Council of Teachers of Mathematics.
- White, D., Donaldson, B., Hodge, A., & Ruff, A. (2013). Examining the effects of math teachers' circles on aspects of teachers' mathematical knowledge for teaching. *International Journal for Mathematics Teaching and Learning*. Retrieved from September 26, 2013, at <http://www.cimt.plymouth.ac.uk/journal/>